Transactions of the Nigerian Association of Mathematical Physics
Volume 1, (November, 2015), **pp 201 – 208**

Effects of Internal Heat Generation and Thermo Diffusion on Convection Heat and Mass Transfer in A Hydromagnetic Flow of A Second Grade Fluid in The Presence of Thermal Radiation and Thermal Diffusion

[1]Olanrewaju, P.O., [2]Abbaz, Z. and [3]Olanrewaju, M.A.

[1]Department of Mathematics and Statistics, Federal University Wukari. Taraba State, Nigeria
[2]Department of Mathematics, The Islamia University of Bahawalpur, Pakistan.
[3]Department of Mathematics, University of Ilorin, Kwara State.

Abstract

An analysis is carried out to study the influence of internal heat generation, thermal and thermo diffusion on convection heat and mass transfer in a hydromagnetic flow of a second grade fluid past a semi-infinite stretching sheet in the presence of thermal radiation. Similarity solutions are obtained using scaling transformations. Using the similarity variables, the governing non-linear partial differential equations are transformed into a set of coupled non-linear ordinary differential equations, which are solved numerically by the shooting iteration technique together with a sixth order Runge–Kutta integration scheme. Numerical calculations of the local skin friction coefficient, the localNusselt number and the local Sherwood number as well as velocity, temperature and concentration profiles of the fluid, are presented for different physical parameters. Most importantantly, the paper correct mistakes in Olajuwon [14] and also extended the work to show the significant of the embedded flow parameters in the flow.

1.0 Introduction

In recent years, the study of non-Newtonian fluid is quite useful because of its variety of applications in various disciplines. Due to the complexity of non-Newtonian fluids there are many models of non-Newtonian fluids. An extensive study of non-Newtonian fluid models is available in the refs [1-10]. To be more specific heat transfer analysis plays an important role in non-Newtonian fluids especially in the handling and processing of coal-based slurries. In some situations, it is not necessary that the fluid viscosity is constant and it may vary with temperature or pressure. For example, in coal slurries the viscosity of the fluid may vary with temperature.

The theoretical study of magnetohydrodynamic (MHD) channel flow has been a subject of great interest due to itswidespread applications in designing cooling systems with liquid metals, MHD generators, accelerators, pumps andflow meters.

Hayat and Abbas [11] examined the heat transfer analysis on the MHD flow of a second grade fluid in a channel with porous medium. Hayat, Abbas, Sajid & Asghar, [12] studied the influence of thermal radiation on MHD flow of a second grade fluid. Hayat, Ahmed, Sajid, & Asghar, [13] investigated the MHD flow of a second grade fluid in a porous channel. Olajuwon [14] discussed the Convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion.

Hayat, Shehzad, Qasim & Obaidat [15] *investigated* Flow of a second grade fluid with convective boundary conditions.Abdul Majeed, Tahira & Zarqa [16] studied Slip Effects on the Flow of a Second Grade Fluid in a Varying Width Channel with Application to Stenosed Artery. Singh and Agarwal [17] examined heat transfer in a second grade fluid over an exponentially stretching sheet through porous Medium with thermal radiation and elastic Deformation under the effect of magnetic field. Baris [18] discussed the flow of a Second-Grade Visco-Elastic Fluid in a Porous Converging Channel.

Olanrewaju and Abbas [19] examined the Corrigendum to "Convection Heat and Mass Transfer in a Hydromagnetic Flow of a Second Grade fluid in the Presence of Thermal Radiation and Thermal Diffusion.

The research work is motivated by the above referenced work. In particular, Olajuwon [14]who discussed the Convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal

Corresponding author: Olanrewaju, P.O., E-mail: oladapo_anu@yahoo.ie, Tel.: +2348138485550

diffusion to include detailed analysis of fluid flow parameters with Newtonian heating which no one has ever studied with the author's knowledge. The goal of the present work is therefore, to investigate the combined effects of details parameters analysis of heat and mass transfer on second grade fluid and to further correct errors in Olajuwon [14] as Olanrewaju & Abbas [19] in there comment on Olajuwon [14].

2.0 Governing Equations

We consider the steady convection heat and mass transfer in a hydromagnetic flow of a second grade fluid past a semi-infinite stretching sheet with the combine effects of internal heat generation, thermal and thermo diffusion in the presence of thermal radiation. Let u and v be the velocity component along the x and y directions, respectively. The surface is maintained at a constant temperature T_w, which is higher than the constant temperature T_∞ of the surrounding and concentration C_w is greater than the constant concentration C_∞. The fluid properties are assumed to be constant. Since the plate is vertically upward, the governing equations of continuity, momentum, energy and concentration for the steady flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\lambda\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right] \tag{2}$$
$$-\frac{\sigma\beta_0^2 u}{\rho}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty),$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{\alpha}{k}\frac{\partial q_r}{\partial y}+\frac{Q}{k}(T-T_\infty)+\frac{\rho D_M K_T}{c_s}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Satisfying the boundary conditions

$$u=Ax,\ v=V,\ T=T_w,\ C=C_w\ \ at\ y=0$$
$$u\to 0,\ \frac{\partial u}{\partial y}\to 0,\ T\to T_\infty, C\to C_\infty\ as\ y\to\infty, t>0 \tag{5}$$

where *u*, *v* are the velocity components in the *x* and *y* directions respectively, υ is the kinematic viscosity, *g* is the acceleration due to gravity, *ρ* is the density, *β* is the coefficient of volume expansion, β^* is the volumetric coefficient of expansion with concentration, *T*, T_w and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, while *C*, C_w and C_∞ are the corresponding concentrations. Also, *α* is the thermal diffusivity, D_m is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, k_T is the thermal diffusion ratio, and c_s is the concentration susceptibility.

Here we introduce the stream function ψ defined as $u=\partial\psi/\partial x$ and $v=-\partial\psi/\partial x$, which identically satisfies Eq. (1).

The radiative heat flux q_r is described by Roseland approximation such that

$$q_r = -\frac{4\sigma^*}{3K}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ^* *and* K are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Following Olajuwon (2011), we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4 \approx 4T_\infty^3 T-3T_\infty^4. \tag{7}$$

Using (7) and (8) in (3), we obtain

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^*}{3K}\frac{\partial^2 T^4}{\partial y^2}. \tag{8}$$

Using the similarity transformations of the form

$$\eta = y\sqrt{\frac{A}{\upsilon}},\ u = Axf',\ v = -\sqrt{A\upsilon}\,f,\quad \theta = \frac{T - T_\infty}{T_f - T_\infty},\ \phi = \frac{C - C_\infty}{C_f - C_\infty}. \quad (9)$$

Applying equation (7) on equations (1)-(6), we obtain

$$f'^2 - ff'' - f''' - K_1\left[2ff''' - ff^{iv} - f''^2\right] + Mf' - Gr\theta - Gc\phi = 0, \quad (10)$$

$$\frac{1}{\Pr}\theta''\left(1 + \frac{4}{3}Rd\right) + f\theta' + Du\phi'' + \lambda_1\theta = 0, \quad (11)$$

$$\frac{1}{Sc}\phi'' + f\phi' + Sr\theta'' = 0 \quad (12)$$

Satisfying the boundary conditions

$$\begin{aligned} &f = f_w,\ f' = 1,\ \theta = 1,\ \phi = 1\ \ at\ \ \eta = 0,\\ &f' = 0,\ \ f'' = 0,\ \ \theta = 0,\ \phi = 0\ \ for\ \eta \to \infty \end{aligned} \quad (13)$$

Where:

$$\begin{aligned} &M = \frac{\sigma B_0^{\,2}}{\rho\upsilon A^{\frac{3}{2}}x}.\quad K_1 = \left(\frac{\upsilon}{A}\right)^{\frac{1}{2}}\frac{\lambda}{A},\quad Gr = \frac{g\beta\lambda}{A^2x}\left(T_f - T_\infty\right), Gc = \frac{g\beta^*}{A^2x}\left(T_f - T_\infty\right)\\ &Rd = \frac{4\sigma^* T_\infty^{\,3}}{\delta\upsilon},\ \Pr = \frac{\upsilon}{\alpha},\ Du = \frac{D_m k_T}{c_s c_\rho \upsilon}\left(\frac{C_f - C_\infty}{T_f - T_\infty}\right),\quad Sc = \frac{\upsilon}{D_m},\\ &Sr = \frac{D_m k_T}{T_m\upsilon}\left(\frac{T_f - T_\infty}{C_f - C_\infty}\right),\ \lambda_1 = \frac{Q}{\rho c_\rho A} \end{aligned} \quad (14)$$

K_1 is the second grade parameter, *M*is the magnetic field parameter, *Gr* is the thermal Grashof number, *Gc* is the solutal Grashof number, *Pr* is the Prandtl number, *Rd* is the radiation parameter
λ_1 *is* the internal heat generation, *Du* is the Dufour number, *Sc* is the Schmitt number, *Sr* is the Soret number.

3.0 Numerical Procedure

The set of Eqs. (10)-(12) together with the boundary conditions (13) have been solved numerically by applying Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta sixth-order integration method. From the process of numerical computation, the skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to $f''(0), -\theta'(0)\ and\ -\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form. The computations have been performed by a program which usesa symbolic and computational computer language MAPLE [31]. A step size of $\Delta\eta = 0.001$ is selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of y_∞ is found to each iteration loop by the assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, to each group of parameters, *Pr, Sc, Sr,* D_u, *M, Rd,* G_T, G_C, K_1, $f_w$and λ_1 is determined when the values of unknown boundary conditions at $\eta = 0$ do not change to successful loop with error less than 10^{-7}.

4.0 Results and Discussions

In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the flow parameters encountered in the problem. To be realistic, the values of the embedded parameters were chosen following Olanrewaju & Abbas [19] Attention is focused on positive values of the buoyancy parameters i.e. Grashof number $G_T > 0$ (which corresponds to the cooling problem) and solutal Grashof number $G_C > 0$ (which indicates that the chemical species concentration in the free stream region is less than the concentration at the boundary surface) and the volumentric heat generation/absorption parameter. The cooling problem is often encountered in engineering applications. In table 1, shows the influenceof embedded fluid flow parameters on the overall flow structure. Figures 1- 8 represent the influence of second grade parameter, magnetic field parameter, thermal Grashof number, solutal Grashof number, thermal radiation parameter, Prandtl number, Dufour number, internal heat generation, Schmitt number, Soret number and the fluid suction/injection on the velocity, temperature and concentration profiles. In table 1, increase in the parameters $K_1, G_r, G_c, Rd, Du\ and\ \lambda_1$ bring a decrease in the Skin-friction. There is an increment in the Skin-friction when the following parameters $M, \Pr, Sc, Sr, and\ f_w$ increases. The Sherwood number (mass transfer) increases as the following parameters $M, Rd, Du, \lambda_1, Sc, Sr, f_w$ increases while it decreases when the following parameters $K_1, G_r, G_c, \Pr$ increases. Similarly, an increase K_1, M, G_r, G_c Pr, f_w parameters leads to a decrease in the rate of heat transfer (Nusselt number)at the surface while an increase in the $R_d, Du, \lambda_1, Sc, Sr$ parameters increases the rate of heat transfer at the surface. Figure 1 depict the velocity profile for various values of the second grade fluid $K_1$for fixed values of other fluid flow parameters. It was observed that increase in this parameter thickens the velocity boundary layer thickness but there is a decrease in the concentration boundary layer in figure 2. In figure 3, we plot the graph of velocity against spanwise coordinate η . It was interesting to note that the velocity boundary layer thickness decrease as magnetic parameter M increases. Figure 4 represents the temperature distribution for various values of magnetic field M. it was observed that the temperature boundary layer thickness thickens as M increases. In figure 5, we plot the graph of velocity against spanwise coordinate η . It was observed that as the radiation parameter Rd increases, the velocity boundary layer thickness thickens. Similar effect was observed in figure 6. The temperature distribution increases as the radiation parameter increases. In figure 7, we observed that as the Dufour number Du increases, the temperature distribution increases which leads to an increase in the temperature boundary layer thickness. Finally, figure 8 depicts the plot of temperature profiles for various values of the internal heat generation λ_1 and other flow parameters were kept constant. It was observed that the temperature boundary layer thickness thickens as the internal heat generation increases across the boundary.

Table 1: Computation showing $F''(0), \theta'(0)$ *and* $\phi'(0)$ for various values of embedded flow parameters.

K_1	M	G_r	G_c	Rd	Pr	Du	λ_1	Sc	Sr	F_w	$-F''(0)$	$-\varphi'(0)$	$\theta'(0)$
1	2	0.9	0.9	0.2	0.71	0.03	1	0.22	0.5	0.5	0.4691859	0.4640089	0.921832
5	2	0.9	0.9	0.2	0.71	0.03	1	0.22	0.5	0.5	0.3938794	0.4318636	0.553996
10	2	0.9	0.9	0.2	0.71	0.03	1	0.22	0.5	0.5	0.3575764	0.4120481	0.286676
1	5	0.9	0.9	0.2	0.71	0.03	1	0.22	0.5	0.5	0.7641985	0.6312553	2.715631
1	10	0.9	0.9	0.2	0.71	0.03	1	0.22	0.5	0.5	1.7593533	0.1368359	-1.07231
1	2	3	0.9	0.2	0.71	0.03	1	0.22	0.5	0.5	0.1094976	0.4066164	0.127894
1	2	5	0.9	0.2	0.71	0.03	1	0.22	0.5	0.5	-0.168165	0.4036781	0.049129
1	2	10	0.9	0.2	0.71	0.03	1	0.22	0.5	0.5	-0.735360	0.4120147	0.232315
1	2	0.9	3	0.2	0.71	0.03	1	0.22	0.5	0.5	0.1783406	0.4052548	0.121649
1	2	0.9	5	0.2	0.71	0.03	1	0.22	0.5	0.5	-0.133285	0.4053579	0.115941
1	2	0.9	10	0.2	0.71	0.03	1	0.22	0.5	0.5	-0.797909	0.4276033	-0.32712
1	2	0.9	0.9	0.5	0.71	0.03	1	0.22	0.5	0.5	0.4242632	0.4785658	0.992129
1	2	0.9	0.9	0.7	0.71	0.03	1	0.22	0.5	0.5	0.4048552	0.4828092	0.998740
1	2	0.9	0.9	0.2	3.0	0.03	1	0.22	0.5	0.5	0.7470419	0.2756324	-0.57285
1	2	0.9	0.9	0.2	3.1	0.03	1	0.22	0.5	0.5	0.7499712	0.2707745	-0.61664
1	2	0.9	0.9	0.2	0.71	0.10	1	0.22	0.5	0.5	0.4618462	0.4699391	0.970877
1	2	0.9	0.9	0.2	0.71	0.50	1	0.22	0.5	0.5	0.4264786	0.4998202	1.222411
1	2	0.9	0.9	0.2	0.71	0.03	2	0.22	0.5	0.5	-0.361216	1.4232846	10.09685
1	2	0.9	0.9	0.2	0.71	0.03	3	0.22	0.5	0.5	-1.174991	3.1547328	27.77152
1	2	0.9	0.9	0.2	0.71	0.03	1	0.62	0.5	0.5	0.4905565	1.1034120	1.248939
1	2	0.9	0.9	0.2	0.71	0.03	1	1.00	0.5	0.5	0.5132586	1.6287182	1.323723
1	2	0.9	0.9	0.2	0.71	0.03	1	1.50	0.5	0.5	0.5340415	2.2678806	1.366827
1	2	0.9	0.9	0.2	0.71	0.03	1	0.22	1.0	0.5	0.4812477	0.5555112	0.877687
1	2	0.9	0.9	0.2	0.71	0.03	1	0.22	1.5	0.5	0.4920749	0.6393315	0.835367
1	2	0.9	0.9	0.2	0.71	0.03	1	0.22	0.5	1.0	0.6524430	0.4423484	0.108243
1	2	0.9	0.9	0.2	0.71	0.03	1	0.22	0.5	1.5	0.7846273	0.4504944	-0.53012
1	2	0.9	0.9	0.2	0.71	0.03	1	0.22	0.5	2.0	0.8595343	0.4929009	-0.95915

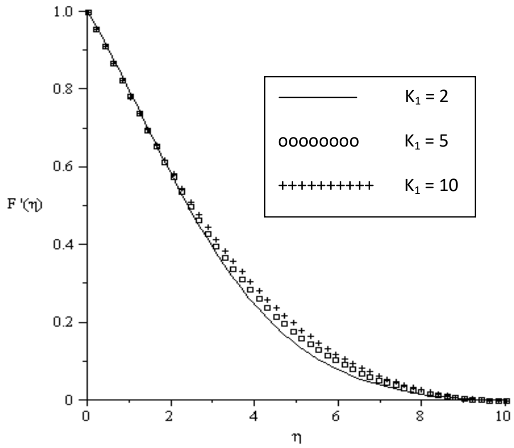

Figure 1: Effects of second grade parameter K on the velocity profile for fixed values of Pr = 0.72, Rd=1, M = 0.1, G_r= 0.9, G_c= 0.9, Sr= 0.5, Du = 0.03. Sc = 0.62, F_w = 0.5, λ_1 = 1

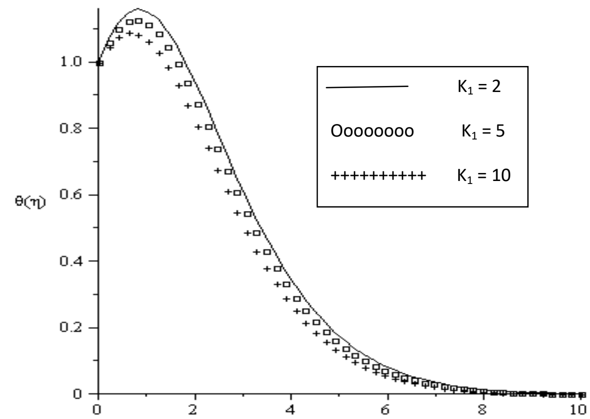

Figure 2: Effects of second grade parameter K on the temperature profile for fixed values of Pr = 0.72, Rd=1, M = 0.1, G_r= 0.9, G_c= 0.9, Sr= 0.5, Du = 0.03. Sc = 0.62, F_w = 0.5, λ_1 = 1

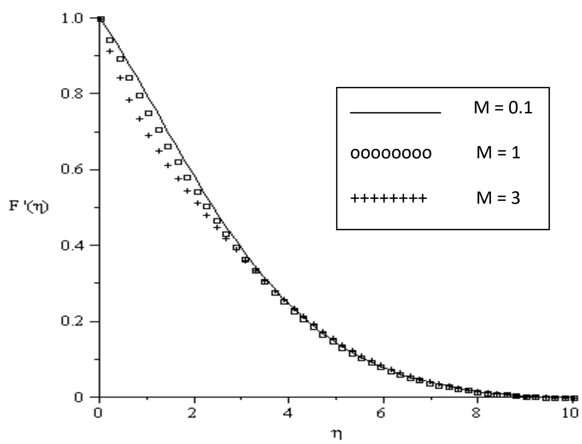

Figure 3: Effects of magnetic field parameter M on the velocity profile for fixed values of Pr = 0.72, Rd=1, K = 2, G_r= 0.9, G_c= 0.9, Sr= 0.5, Du = 0.03. Sc = 0.62, F_w = 0.5, λ_1 = 1

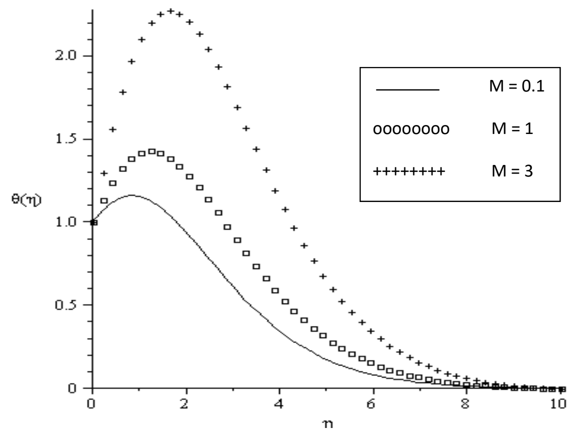

Figure 4: Effects of magnetic field parameter M on the temperature profile for fixed values of Pr = 0.72, R_d=1, K = 2, G_r= 0.9, G_c= 0.9, Sr= 0.5, Du = 0.03. Sc = 0.62, F_w = 0.5, λ_1 =

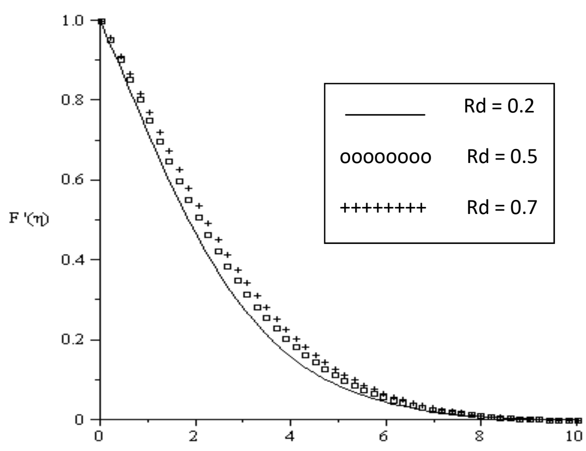

Figure 5: Effects of thermal radiation R_d on the velocity profile for fixed values of Pr = 0.72, M = 0.1, K = 2, G_r= 0.9, G_c= 0.9, Sr= 0.5, Du = 0.03. Sc = 0.62, F_w = 0.5, λ_1 = 1

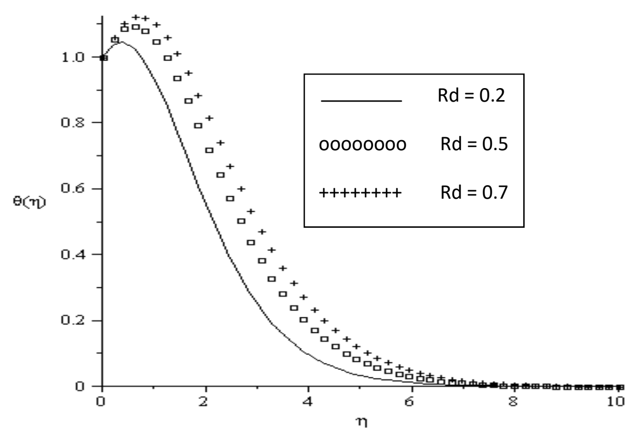

Figure 6: Effects of thermal radiation R_d on the temperature profile for fixed values of Pr = 0.72, M = 0.1, K = 2, G_r= 0.9, G_c= 0.9, Sr= 0.5, Du = 0.03. Sc = 0.62, F_w = 0.5, λ_1 = 1

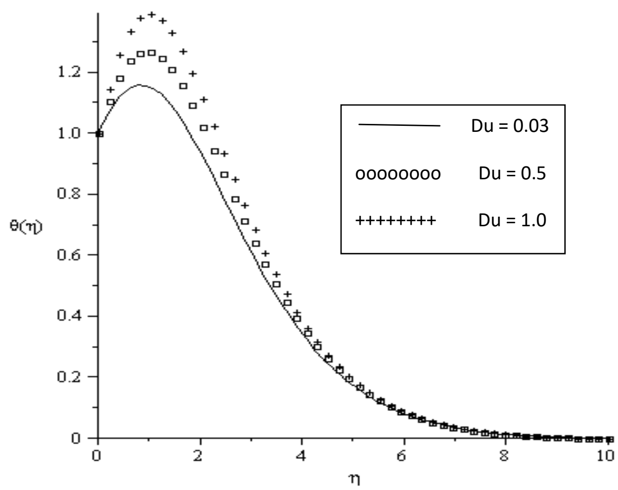

Figure 7: Effects of Dufour parameter Du on the temperature profile for fixed values of Pr = 0.72, M = 0.1, K = 2, G_r= 0.9, G_c= 0.9, Sr= 0.5, Rd = 1. Sc = 0.62, F_w = 0.5, λ_1 = 1

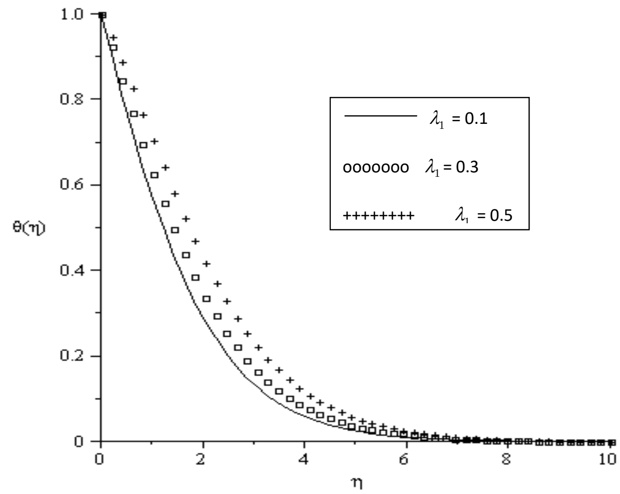

Figure 8: Effects of internal heat generation parameter λ on the temperature profile for fixed values of Pr = 0.72, M = 0.1, K = 2, G_r= 0.9, G_c= 0.9, Sr= 0.5, Rd = 1. Sc = 0.62, F_w = 0.5, Du = 0.03

5.0 Conclusion

We have studied theoretically the influence of internal heat generation, thermal and thermo diffusion on convection heat and mass transfer in a hydromagnetic flow of a second grade fluid past a semi-infinite stretching sheet in the presence of thermal radiation.Similarity solutions are obtained using scaling transformations. Using the similarity variables, the governing non-linear partial differential equations are transformed into a setof coupled non-linear ordinary differential equations, which are solved numerically by the shooting iterationtechnique together with a sixth order Runge–Kutta integration scheme.Numerical calculations of the local skin friction coefficient, the localNusselt number and the local Sherwood number as well as velocity, temperature and concentration profiles ofthe fluid, are presented for different physical parameters. Most importantantly, the paper correct mistakes in Olajuwon [14] and also extended the work to show the significant of the embedded flow parameters in the flow. The following findings were discovered as follows:

- The second grade fluid parameter has greater influence on the velocity and concentration distribution. Increasing K_1 thickens the velocity and concentration boundary layer thickness.
- Increase in the internal heat generation λ_1 parameter thickens the temperature boundary layer thickness across the channel.
- The thermal radiation parameter Rd has greater influence on the velocity and temperature profiles. There was a sudden jump close to the wall on temperature boundary layer thickness as radiation parameter increases. Generally, increase in the so called parameter thickens the velocity and temperature boundary layer thickness across the channel.
- Dufour number Du has a great influence on the thermal boundary layer. The temperature boundary layer increases in thickness as the Dufour number increases.
- It was seen that the magnetic field parameter step down the rate of flow but thickens the thermal boundary layer thickness

6.0 References

[1] S Nadeem and NS Akbar. Influence of heat transfer on peristaltic transport of a Johnson Segalman fluid in an inclined asymmetric channel.*Comm. Nonlinear Sci. Numer. Simul*. 2010;**15**, 2860-77.

[2] R Ellahi and A Riaz. Analytical solution for MHD flow in a third grade fluid with variable viscosity.*Math. Comput. Model*.2010;**52**,1783-93.

[3] S Nadeem and S Akram. Influence of inclined magnetic field on the peristaltic flow of a Williamson fluid model in an inclined symmetric and asymmetric channel.*Math. Comput. Model*.2010; **52**,107-19.

[4] R Ellahi. Exact and numerical solutions for nonlinear differential equation of Jeffrey-Hamel flow.*Int. J. Industrial Math*.2011;**3**, 1-7.

[5] R Ellahi. Exact solutions of flows of an Oldroyd 8-constant fluid with nonlinear slip conditions.*Zeitschrift Fur Naturforschung A* 2010; **65**, 1081-6.

[6] S Nadeem and NS Akbar. Peristaltic flow of Sisko fluid in a uniform inclined tube, *ActaMechanica Sinica*. 2010; **26**,675-83.

[7] S Nadeem, NS Akbar and MY Malik. Exact and numerical solutions of a micropolar fluid in a vertical Annulus.*Numer.Meth.Part.Differ.Equat.*2010;**26**, 1660-74.

[8] D Tripathi. Peristaltic transport of a viscoelastic fluid in a channel.*Acta Astronautica*2011;**68**, 1379-85.

[9] NS Akbar and S Nadeem. Analytical and numerical analysis of Vogel's model of viscosity on the peristaltic flow of Jeffrey fluid.*J.Aerospace Eng.*2012;**25**,64-71.

[10] NS Akbar and S Nadeem.Peristaltic flow of a Phan-Thien-Tanner nanofluid in a diverging tube.*Heat Tran. Asian Res.*2012; **41**,10-22.

[11] Hayat, T. & Abbas, Z. (2008). Heat Transfer Analysis on the MHD Flow of a Second Grade Fluid in a Channel with Porous Medium, Chaos, Solitons and Fractals 38: 556–567.

[12] Hayat, T., Abbas, Z., Sajid, M. & Asghar, S. (2007). The Influence of Thermal Radiation on MHD Flow of a Second Grade fluid, *International Journal of Heat and Mass Transfer* 50: 931–941.

[13] Hayat, T., Ahmed Naveed, Sajid, M. & Asghar, S. (2007). On the MHD flow of a second Grade Fluid in a Porous Channel, Computers and Mathematics with Applications 54: 407–414.

[14] Olajuwon, B.I. (2011). Convection Heat and Mass Transfer in a Hydromagnetic Flow of a second grade fluid in the presence of thermal radiation and thermal diffusion, *Int. J. Heat Mass Transf.* Vol. 38, 3: 377–382.

[15] Hayat Tasawar, Shehzad, S. A., Qasim Muhammad & Obaidat Saleem (2011). Flow of a Second Grade Fluid with Convective Boundary Conditions, *Thermal Science*, Vol. 15, Suppl. 2: S253-S261.

[16] Abdul Majeed Siddiqui, Tahira Haroon, & Zarqa Bano, (2013). Slip Effects on the Flow of a Second Grade Fluid in a Varying Width Channel with Application to Stenosed Artery, *Universal Journal of Applied Mathematics 1*(2): 72-81.

[17] Singh, V. & Agarwal, S. (2012). Heat transfer in a second grade fluid over an exponentially stretching sheet through porous medium with thermal radiation and elastic deformation under the effect of magnetic field, Int. J. of Applied Math and Mech. 8(4): 41-63.

[18] Baris Serdar (2003). Flow of a Second-Grade Visco-Elastic Fluid in a Porous Converging Channel, *Turkish J. Eng. Env. Sci.* 27: 73 – 81.

[19] Olanrewaju, P.O., & Abbas, Z., (2014). Corrigendum to "Convection Heat and Mass Transfer in a Hydromagnetic Flow of a Second Grade fluid in the Presence of Thermal Radiation and Thermal Diffusion" (*Int. J. Heat Mass Transf.* 38 (3) (2011) 377–382), *Int. J. Heat Mass Transf.* 51: 59-60

[20] Heck, A. (2003). *Introduction to Maple*, 3rd ed., Springer-Verlag, New York