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Computational Results on Quadratic Functional Model for the Tokens Of Nuclear Safety

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Abstract

In this work, Nuclear Reactor safety was modeled inform of quadratic functional. The nuclear tokens are structured and used as elements of the control matrix operator in our quadratic functional. The numerical results obtained through Conjugate Gradient Method (CGM) algorithm identify the optimal level of safety required for Nuclear Reactor construction at any particular situation.

1.0 Introduction

Nuclear reactors accidents occur when the coolant ceases to work, the reactor will be over heated and produced excess heat in form of steam. Most of the internal component of the reactors are made from zirconium in zircalloy cladding used in fuel rods oxidizes in reaction with steam to produce zirconium oxide and hydrogen [1]. When mixed with air, hydrogen is flammable and its detonation will destroy the containment structure which house the reactor. As a result of this, radiation is release to the surrounding causing environmental hazard [2-5]. This work derived the equations for the rate of heat of different reactors from energy balance equations. These equations are structured in parametric form to obtain the basis of the mathematical model solvable by the Conjugate Gradient Method (CGM) algorithm. The obtained numerical results generate the minimal disaster associated with nuclear reactors.

2.0 Energy Balance

The following results were obtained through Energy Balance Equations. See [6]. For the purpose of our study, we shall arrange the equations in terms of the rate of heat.

2.1 The Energy Balance for Chemical Reactors

Let us consider arbitrary reactor volume element. From the conservation of energy for a reactor system we obtain the following:

$$\begin{pmatrix} Rate of Energy \\ accumulated \end{pmatrix} = \begin{pmatrix} Rate of Energy \\ of inflow \end{pmatrix} - \begin{pmatrix} Rate of Energy \\ of outflow \end{pmatrix} + \begin{pmatrix} Rate of heat added \\ to the system \end{pmatrix} + \begin{pmatrix} Rate of work done \\ on the system \end{pmatrix} \quad (1)$$

Equation (1) can be represented mathematically as:

$$\frac{dE}{dt} = m_0 \hat{E}_0 - m_1 \hat{E}_1 + \dot{Q} + \dot{W} \quad (2)$$

where $\hat{E}$ mean energy per unit mass, m_0 is the mass inflow, m_1 is the mass outflow and $\dot{Q}$ is the rate of heat.

The total rate of work done on a reactor system is expressed as follows:

$$\dot{W} = \dot{W}_f - \dot{W}_s + \dot{W}_b \quad (3)$$

where $\dot{W}$ =total rate of work done

$\dot{W}_f$ =rate of work done by flowstream

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$\dot{W}_s$ =rate of work done by shaft

$\dot{W}_b$ =rate of work done by boundary

The Rate of work done flowstream can be represented by $\dot{W}_f$ such that

$$\dot{W}_f = v_0 A_0 P_0 - v_1 A_1 P_1 = Q_0 P_0 - Q_0 P_0 \tag{4}$$

where A_0 =area of reactor (inflow), A_1 =area of reactor (outflow), v_0 =inflow volume of reactor, v_1 =outflow volume of reactor, P_0 = inflow pressure, P_1 = outflow Pressure, $Q_{(0)}$ =flowrate (in), Q_1 =flowrate (out), m =mass ρ_0 =inflow density, ρ_1 = outflow density and ρ = general density, also

$$Q = \frac{m}{\rho} \tag{5}$$

Substituting (5) in (4) and using the result in (3), we obtain

$$\dot{W} = m_0 \frac{P_0}{\rho_0} - m_1 \frac{P_1}{\rho_1} + \dot{W}_s + \dot{W}_b \; h \tag{6}$$

The energy terms of total energy composed in Internal U, Kinetic K and Potential energy ϕ is expressed as:

$$E = U + K + \phi \tag{7}$$

Substituting (7) in (2), we obtain

$$\frac{d}{dt}(U + K + \phi) = m_0(\hat{U} + \hat{K} + \hat{\phi})_0 - m_1(\hat{U} + \hat{K} + \hat{\phi})_1 + \dot{Q} + \dot{W}_s + \dot{W}_b \tag{8}$$

but in chemical reactors, only the internal energy is considered with the enthalphy $H = U + PV$ per unit mass, hence (8) becomes

$$\frac{d}{dt}U = m_0 \hat{H}_0 - m_1 \hat{H}_1 + \dot{Q} + \dot{W}_s + \dot{W}_b \tag{9}$$

2.2 The Batch Reactor

The batch reactors have no flowstream (i.e. $m_0 \hat{H}_0 - m_1 \hat{H}_1 = 0$).

Therefore, equation (9) in terms of rate of heat becomes

$$\dot{Q}_{11} = \frac{du}{dt} - \dot{W}_s - \dot{W}_b \tag{10}$$

Neglecting the work done by stirrer because the mixture is not highly viscous,
so the stirring operation does not draw significant power, (10) yields

$$\dot{Q} = \frac{du}{dt} - \dot{W}_b \tag{11}$$

and we know that $\dot{W}_b = -P \frac{dV_R}{dt}$, hence (11) becomes

$$\dot{Q}_{12} = \frac{du}{dt} + P \frac{dV_R}{dt} \tag{12}$$

For Batch reactor in terms of enthalpy, we have

$$H = U + PV_R \tag{13}$$

Taking the differential of (13) for $V = V_R$ and substituting in (12), we obtain

$$\dot{Q}_{13} = \frac{dH}{dt} - V_R \frac{dP}{dt} \tag{14}$$

We now consider enthalpy as a function of temperature T, pressure P and number of moles n_j, and express its differentials as;

$$\frac{dH}{dt}=\left(\frac{\partial H}{\partial T}\right)_{P,n_j}\frac{dT}{dt}+\left(\frac{\partial H}{\partial P}\right)_{T,n_j}\frac{dP}{dt}+\sum_j\left(\frac{\partial H}{\partial n_j}\right)_{T,P,n_k}\frac{dn_j}{dt} \tag{15}$$

The first partial derivative is the definitions of the heat capacity, C_P, that is

$$C_P=V_R\rho\hat{C}_P \tag{16}$$

The second partial derivative can be expressed as

$$\left(\frac{\partial H}{\partial P}\right)_{T,n_j}=V-T\left(\frac{\partial V}{\partial T}\right)_{P,n_j}=V\left(1-\alpha T\right) \tag{17}$$

where $\alpha=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P,n_j}$ is the coefficient of expansion of mixture.

The final partial derivatives are the partial molar enthalpies, $\bar{H}_j$

$$\left(\frac{\partial H}{\partial n_j}\right)_{T,P,n_k}=\bar{H}_j \tag{18}$$

Substituting (16), (17) and (18) in (15) and using the result in (14), we obtain

$$\dot{Q}_{14}=V_R\rho\hat{C}_P\frac{dT}{dt}-\alpha TV_R\frac{dP}{dt}+\sum_j\bar{H}_j\frac{dn_j}{dt} \tag{19}$$

But the material balance for batch reactor is

$$\frac{dn_j}{dt}=R_jV_R=\sum_{i=1}^{n_r}V_{ij}r_iV_R \qquad j=1,\cdots,n_s \tag{20}$$

where V_{ij} is the stoichiometric coefficient for species j and reaction i, R_j is the production rate for j th species and r_i is the reaction rate for i th reaction, and the heat of reaction is

$$\Delta H_{Ri}=\sum_j V_{ij}\bar{H}_j \tag{21}$$

Substituting (20) and (21) in (19) we obtain

$$\dot{Q}_{15}=V_R\rho\hat{C}_P\frac{dT}{dt}-\alpha TV_R\frac{dP}{dt}+\sum_j\Delta H_{Ri}r_iV_R \tag{22}$$

The constant-pressure batch reactor is the incompressible-fluid and for $\frac{dP}{dt}=0$ then equation (22), becomes

$$\dot{Q}_{16}=V_R\rho\hat{C}_P\frac{dT}{dt}+\sum_j\Delta H_{Ri}r_iV_R \tag{23}$$

If the heat removal is manipulated to maintain constant reactor temperature, the time derivative in equation (23) vanishes leaving

$$\dot{Q}_{21}=\Delta H_RrV_R \tag{24}$$

When $C_A=$ concentration of species A, $k=$ reaction rate constant, $r=kC_A^2$ and H_R is the enthalpy change on reaction then equation (24) becomes

$$\dot{Q}_{22}=\Delta H_RkC_A^2V_R \tag{25}$$

For the constant-volume batch reactor, we considered the pressure as function of temperature, volume and number of moles, and also expressed its differentials as:

$$\frac{dP}{dt}=\left(\frac{\partial P}{\partial T}\right)_{V,n_j}\frac{dT}{dt}+\left(\frac{\partial P}{\partial V}\right)_{T,n_j}\frac{dV}{dt}+\sum_j\left(\frac{\partial P}{\partial n_j}\right)_{T,V,n_k}\frac{dn_j}{dt} \tag{26}$$

For reactor operation at constant volume, $\frac{dV}{dt}=0$, and forming time derivatives, just as we did in (15) to (17) and substituting into equation (19) gives

$$\dot{Q}=\left[V_R\rho\hat{C}_P-\alpha TV_R\left(\frac{\partial P}{\partial T}\right)_{V,n_j}\right]\frac{dT}{dt}+\sum_j\left[\bar{H}_j-\alpha TV_R(\frac{\partial P}{\partial n_j})_{T,V,n_k}\right]\frac{dn_j}{dt} \tag{27}$$

Note that the first term in brackets is the total constant-volume heat capacity, that is

$$C_v=V_R\rho\hat{C}_P-\alpha TV_R\left(\frac{\partial P}{\partial T}\right)_{V,n_j} \tag{28}$$

Substitution (28) and the material balance in (20), yields the rate of heat for the energy balance of the Constant-Volume batch Reactor. That is

$$\dot{Q}_{23}=V_R\rho\hat{C}_v\frac{dT}{dt}+\sum_j\left[\Delta H_{Ri}-\alpha TV_R\sum_j V_{ij}(\frac{\partial P}{\partial n_j})_{T,V,n_k}\right]r_iV_R \tag{29}$$

If we consider a constant volume-ideal gas, where $\alpha T=1$ and $\left(\frac{\partial P}{\partial n_j}\right)_{T,V,n_k}=\frac{RT}{V}$.

Substituting these into (29) gives

$$\dot{Q}_{24}=V_R\rho\hat{C}_v\frac{dT}{dt}+\sum_j\left[\Delta H_{Ri}-RT\bar{V}_i\right]r_iV_R \tag{30}$$

Where

$$\bar{V}_i=\sum_j V_{ij} \tag{31}$$

2.3 The Continuous Stirred Tank Reactor (CSTR)

In order to describe the dynamic operation of a CSTR, the energy balance equation must be developed. The CSTR has flowstream, hence using the equations (8)

$$\frac{d}{dt}(U+K+\phi)=m_0(\hat{U}+\hat{K}+\hat{\rho})_0-m_1(\hat{U}+\hat{K}+\hat{\phi})_1+\dot{Q}+\dot{W}_s+\dot{W}_b \tag{32}$$

As in (9) only the internal energy is considered. The out flow stream is flowing out of a well-mixed reactor, thus, the CSTR rate of heat equation using (32) is

$$\dot{Q}_{25}=\frac{du}{dt}-Q_f\rho_f\hat{H}_f+Q\rho\hat{H}-\dot{W}_s-\dot{W}_b \tag{33}$$

where Q_f =volumetric flow rate,

ρ_f =flow density,

$\bar{H}_f$ =flow enthalpy,

C_{jf} =flow concentration with component j and

Q =flow rate.

As before, if sharf work is neglected for the CSTR, equation (33) becomes

$$\dot{Q}_{26}=\frac{du}{dt}-Q_f\rho_f\hat{H}_f+Q\rho\hat{H}+P\frac{dV_R}{dt} \tag{34}$$

and if the enthalpy is considered, we obtain

$$\dot{Q}_{31}=\frac{dH}{dt}-V_R\frac{P}{dt}-Q_f\rho_f\hat{H}_f+Q\rho\hat{H} \tag{35}$$

We consider the change in enthalpy of the continuous stirred tank reactor (CSTR) as a function of temperature, pressure and number of moles, and express its differentials as;

$$\frac{dH}{dt} = V_R \rho \hat{C}_P \frac{dT}{dt} + (1-\alpha T) V_R \frac{dP}{dt} + \sum_j \bar{H}_j \frac{dn_j}{dt} \tag{36}$$

and substituting into equation (35) gives

$$\dot{Q}_{32} = V_R \rho \hat{C}_P \frac{dT}{dt} - \alpha T V_R \frac{dP}{dt} + \sum_j \bar{H}_j \frac{dn_j}{dt} - Q_f \rho_f \hat{H}_f + Q \rho \hat{H} \tag{37}$$

The material balance for the CSTR is

$$\frac{dn_j}{dt} = Q_f C_{jf} - Q C_j + \sum_i v_{ij} r_i V_R \tag{38}$$

After substituting (38) in (37) and re-arrangement yields

$$\dot{Q}_{33} = V_R \rho \hat{C}_P \frac{dT}{dt} - \alpha T V_R \frac{dP}{dt} + \sum_j \Delta H_{Rj} r_i V_R - \sum_j C_{if} Q_f (\bar{H}_{jf} - \bar{H}_j) \tag{39}$$

The equation of rate of heat for constant-pressure in CSTR that is Incompressible fluidand its mean in equation (39) is $\frac{dp}{dt} = 0$ and hence we have

$$\dot{Q}_{34} = V_R \rho \hat{C}_P \frac{dT}{dt} + \sum_i \Delta H_{Ri} r_i V_R - \sum_j C_{if} Q_f (\bar{H}_{jf} - \bar{H}_j) \tag{40}$$

From equation (40), we obtained the equation of rate of heat for constant-volume in CSTR as follows:

$$\dot{Q}_{34} = V_R \rho \hat{C}_P \frac{dT}{dt} + \sum_i \Delta H_{Ri} r_i V_R - \sum_j C_{if} Q_f (\bar{H}_{jf} - \bar{H}_j) \tag{41}$$

Also, from equation (41), the equation of rate of heat for ideal gas is:

$$\dot{Q}_{36} = V_R \rho \hat{C}_V \frac{dT}{dt} + \sum_i \left(\Delta H_{Ri} - RT\bar{V}_i\right) r_i V_R - \sum_j C_{jf} Q_f (\bar{H}_{jf} - \bar{H}_j) - RT \sum_j (C_{jf} Q_f - C_j Q) \tag{42}$$

For steady state constant, we have $\hat{C}_P$, $P = P_f$ and

$$\bar{H}_{jf} - \bar{H}_j = \bar{C}_{ps} (T_f - T) \tag{43}$$

If we re-arrange equation (39) in the form

$$V_R \rho \hat{C}_P \frac{dT}{dt} - \alpha T V_R \frac{dP}{dt} = \dot{Q} - \sum_j \Delta H_{Rj} r_i V_R + \sum_j C_{if} Q_f (\bar{H}_{jf} - \bar{H}_j) \tag{44}$$

By setting the Right hand side of (44) equals zero and substituting (43) in the result gives

$$\dot{Q}_{41} = \sum_j \Delta H_{Rj} r_i V_R - \sum_j Q_f \rho_f \hat{C}_f (T_f - T) \tag{45}$$

The heat removal rate of CSTR required bringing CSTR reactor out-flow stream from final temperature T_f to temperature T and is given (from 45) by

$$\dot{Q}_{42} = Q_f \rho_f \hat{C}_P \Delta T \tag{46}$$

2.4 The Semi-Batch Reactor

The development of the semi-batch reactor energy balance follows directly from the CSTR energy balance derivation of the rate of heat by setting $Q = 0$. The main results in this paper are therefore summarized below:

Neglecting the Kinetic Energy in Equation 33 of the CSTR, when $Q = 0$, we obtain

$$\dot{Q}_{43} = \frac{du}{dt} - Q_f \rho_f \hat{H}_f - \dot{W}_s - \dot{W}_b \tag{47}$$

Also, by neglecting the Sharf work and consider the Enthalpy when $Q = 0$ in (34) and (35) yields

$$\dot{Q}_{44} = \frac{du}{dt} + P \frac{dV_R}{dt} - Q_f \rho_f \hat{H}_f \tag{48}$$

and if the enthalpy is used, we obtain

$$\dot{Q}_{45} = \frac{dH}{dt} - V_R \frac{P}{dt} - Q_f \rho_f \hat{H}_f \tag{49}$$

By setting $Q = 0$ in equations (37) and (39) respectively, we have the enthalpy change of semi-batch reactor as

$$\dot{Q}_{46} = V_R \rho \hat{C}_P \frac{dT}{dt} - \alpha T V_R \frac{dP}{dt} + \sum_j \bar{H}_j \frac{dn_j}{dt} - Q_f \rho_f \bar{H}_f \tag{50}$$

and

$$\dot{Q}_{51} = V_R \rho \hat{C}_P \frac{dT}{dt} - \alpha T V_R \frac{dP}{dt} + \sum_i \Delta H_{Ri} r_i V_R - \sum_j C_{jf} Q_f (\bar{H}_{jf} - \bar{H}_f) \tag{51}$$

The constant pressure semi-batch reactor is the incompressible-fluid batch reactor and in equation (51) when $\frac{dP}{dt} = 0$, we obtain

$$\dot{Q}_{52} = V_R \rho \hat{C}_P \frac{dT}{dt} + \sum_i \Delta H_{Ri} r_i V_R - \sum_j C_{jf} Q_f (\bar{H}_{jf} - \bar{H}_f) \tag{52}$$

For steady state semi-batch reactor when $\hat{C}_P.$ is constant, we have

$$\dot{Q}_{53} = V_R \rho \hat{C}_P \frac{dT}{dt} + \sum_i \Delta H_{Ri} r_i V_R - Q_f \rho_f \hat{C}_P (T_f - T) \tag{53}$$

The equation is derived from the energy balance equation for Plug-flow reactor (PFR) single phase for rate of heat, and is given by:

$$\dot{Q}_{54} = Q \rho \hat{C}_P \frac{dT}{dV} + Q(1 - \alpha T) \frac{dP}{dV} + \sum_i \Delta H_{Ri} r_i \tag{54}$$

Neglecting pressure drop or ideal gas for PFR and from (54), for an Ideal gas $\alpha T = 1$ we have

$$\dot{Q}_{55} = Q \rho \hat{C}_P \frac{dT}{dV} + \sum_i \Delta H_{Ri} r_i \tag{55}$$

The rate of heat equation of PFR for Incompressible fluid is obtain by setting $\alpha T \frac{dP}{dV} = 0$ in equation (54)

$$\dot{Q}_{56} = Q \rho \hat{C}_P \frac{dT}{dV} + Q \frac{dP}{dV} + \sum_i \Delta H_{Ri} r_i \tag{56}$$

The remaining six existing equations related to the rate of heat of a reactor for temperature of heat transfer medium T_a are as stated below:

$$\dot{Q}_{61} = -\frac{k(T)}{1 + k(T)\tau} C_{Af} \Delta H_R \tag{57}$$

$$\dot{Q}_{62} = \frac{C_{PS}}{\tau} (T - T_f) \tag{58}$$

$$\dot{Q}_{63} = \frac{2}{R} \cup^O (T_a - T) \tag{59}$$

$$\dot{Q}_{64} = \cup^O 2\pi R \Delta_Z (T_a - T) \tag{60}$$

$$\dot{Q}_{65} = K_1 [C_A - \frac{1}{k_1} (C_{Af} - C_A)] \Delta H_R V_R - Q_f \rho \hat{C}_P (T_f - T) \tag{61}$$

$$\dot{Q}_{66} = \frac{d}{dV} (Q \rho \hat{H}) \tag{62}$$

The equations derived above from the energy balance equation of chemical reactors [7] are thirty; namely: (10), (12), (14), (19), (22-25), (29-30), (33-35), (37) , (39-42) and (45-56). These equations with the six existing equations, namely (57)-(62),

were structured into mathematical model in form of quadratic functional. The model with some given existing nuclear tokens were solved by the Conjugate Gradient Method algorithm, with MATLAB as a support soft-ware.

3.0 The Gradient Method (CGM) Algorithm

The CGM algorithm was originally developed by Hestenes and Stiefel [8] to minimize and solve problems in quadratic functional of the form:

$$f(X) = f_0 + \langle a, X \rangle_H + \frac{1}{2}\langle X, AX \rangle_H \tag{63}$$

Where f_0, is a constant in H, x is a vector in H. A is a positive definite, symmetric and constant matrix operator.

It has a well worked out theory with an elegant convergence profile. No approximation is used in the proving its convergency.

3.1 Property of Conjugate Gradient Method (CGM)

Algorithm

Some of the several properties of CGM are:

i. It has a quadratic convergence property that is for a quadratic functional on an n-dimensional Hilbert space, it converges in at most n steps.

ii. It requires a relatively small increase in computer time per iteration and memory space. iii. It has a well worked out theory.

3.2 Algorithm

The first element of the descent sequence x_0 is simply guessed. The remaining members of the sequence are then found as follows:

$$P_0 = -g_0 = -(a + A_0) \tag{64}$$

$$X_{i+1} = X_i + \alpha_i P_i \qquad \alpha_i = \frac{\langle g_i, g_i \rangle}{\langle P_i, AP_i \rangle} \tag{65}$$

$$g_{i+1} = g_i + \alpha_i P_i \tag{66}$$

$$P_{i+1} = -g_{i+1} + \beta_i P_i \qquad \beta_i = \frac{\langle g_{i+1}, g_{i+1} \rangle}{\langle g_i, g_i \rangle} \tag{67}$$

Where g_i is the gradient at the ith element of the descent sequence X_i

It has been proved that the algorithm converges at most, in n iteration in a well posed problem and the convergence rate is given as:

$$E(x_n) = \left\{ \frac{1 - \frac{m}{M}}{1 + \frac{m}{M}} \right\}^{2n} E(x_0) \tag{68}$$

Where m and M are smallest and spectrums of matrix A respectively. That is, for an n dimensional problem, the algorithm will converge in at most n iterations [9].

4.0 Computational Results

Our model is:

$$f(X) = f_0 + \langle a, X \rangle_H + \frac{1}{2}\langle X, AX \rangle_H \tag{69}$$

where $X \in \Re^6$ i.e, $X = (x_1 x_2 x_3 x_4 x_5 x_6)^T$, $a = (111111)^T$, $f_0 = 1$ and

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{pmatrix}$$

where

a_{ij} = values of the rate of heat, $i = 1,2,3,4,5,6$; and $j = 1,2,3,4,5,6$

Numerical values are now calculated for our parameters or tokens. In all cases our initial guess is 0 vector that is $x_0 = (000000)^T$ and the results are as shown below:

Problem 1 (For arbitrary tokens)

$$\mathbf{A} = \begin{pmatrix} 360 & 2 & 0 & 173 & 173 & 173 \\ 10 & 208 & 32 & 37 & 34 & -2 \\ 3 & 173 & 173 & 16 & 9 & -3 \\ 2 & 0 & 173 & 173 & 173 & 173 \\ 0 & 1 & 0 & -2 & 113 & -2 \\ -10 & 0 & 1 & 1 & 5 & 108 \end{pmatrix}$$

Problem 2 (For arbitrary tokens)

$$\mathbf{A} = \begin{pmatrix} 108.422 & 2.0 & 0.4032 & 173.16 & 173.16 & 173.16 \\ 10.316 & 113.1035 & 31.5015 & 36.5015 & 34.4964 & -1.7986 \\ 3. & 173.151 & 173.169 & 15.813 & 9. & -2.9065 \\ 2.0935 & 0.4967 & 173.151 & 173.2535 & 173.151 & 173.151 \\ 0.0003 & 0.6565 & -0.1720 & -1.7986 & 207.6612 & -1.7986 \\ -10.4951 & 0.0201 & 0.8280 & 0.8280 & 5.3300 & 359.991 \end{pmatrix}$$

Problem 3 (For nuclear tokens)

$$\mathbf{A} = \begin{pmatrix} 59.9559 & 4.0000 & -24.5726 & 4.0935 & 0.8280 & -0.3726 \\ 4.0000 & 52.9559 & -24.5816 & -24.5636 & -31.7636 & -24.5726 \\ -24.5726 & -24.5816 & 38.5015 & -1.0887 & 3.0935 & 3.5910 \\ 4.0935 & -24.5636 & -1.0887 & 38.5015 & 9.0000 & -0.3726 \\ 0.8280 & -31.7636 & 3.0935 & 9.0000 & 37.5015 & 0.0845 \\ -0.3726 & -24.5726 & 3.5910 & -0.3726 & 0.0845 & 9.9289 \end{pmatrix}$$

Problem 4 (For nuclear tokens)

$$\mathbf{A} = \begin{pmatrix} 80.4469 & 4.0000 & -24.5726 & 4.0935 & 0.8280 & -0.3726 \\ 4.0000 & 59.9559 & -24.5816 & -24.5636 & -31.7636 & -24.5726 \\ -24.5726 & -24.5816 & 52.9559 & -1.0887 & 3.0935 & 3.5910 \\ 4.0935 & -24.5636 & -1.0887 & 38.5015 & 9.0000 & -0.3726 \\ 0.8280 & -31.7636 & 3.0935 & 9.0000 & 38.5015 & 9.9289 \\ -0.3726 & -24.5726 & 3.5910 & -0.3726 & 9.9289 & 37.5015 \end{pmatrix}$$

4.1 Tables of Results

The following Tables are Table 1 for Problem 1, Table 2 for problem 2, Table 3 for problem 3 and Table 4 for problem 4 respectively.

Table 1: Generated from Problem 1

Number of Iteration	Minimizing vectors *X*	Objective function	Gradient
1	x_1 to x_6 = nill	nill	2.449
2	x_1 to x_6 = nill	0.99	1.704
3	x_1 to x_6 = nill	0.99	5.016
4	x_1 to x_6 = nill	1.01	9.724
5	x_1 to x_6 = nill	1.06	14.045
6	x_1 to x_6 = nill	1.13	17.715
7	x_1 to x_6 = nill	1.22	20.808

8	x_1 to x_6 = nill	1.33	23.440
9	x_1 to x_6 = nill	1.45	25.713
10	x_1 to x_6 = nill	1.58	27.704
11	x_1 to x_6 = nill	1.73	29.474
12	x_1 to x_6 = nill	1.89	31.067
13	x_1 to x_6 = nill	2.06	32.519
14	x_1 to x_6 = nill	2.23	33.855
15	x_1 to x_6 = nill	2.42	35.097
16	x_1 to x_6 = nill	2.61	36.260
17	x_1 to x_6 = nill	2.81	37.358
18	x_1 to x_6 = nill	3.02	38.401
19	x_1 to x_6 = nill	3.24	39.397
20	x_1 to x_6 = nill	3.46	40.353
21	x_1 to x_6 = nill	3.68	41.275
22	x_1 to x_6 = nill	3.91	42.167
23	x_1 to x_6 = nill	4.15	43.032
24	x_1 to x_6 = nill	4.39	43.873
25	x_1 to x_6 = nill	4.64	44.694
26	x_1 to x_6 = nill	4.89	45.495
27	x_1 to x_6 = nill	5.15	46.279
28	x_1 to x_6 = nill	5.41	47.046
29	x_1 to x_6 = nill	5.68	47.799
30	x_1 to x_6 = nill	5.95	48.537
31	x_1 to x_6 = nill	6.22	49.261
32	x_1 to x_6 = nill	6.50	49.973
33	x_1 to x_6 = nill	6.78	50.671
34	x_1 to x_6 = nill	7.06	51.358
35	x_1 to x_6 = nill	7.35	52.033
36	x_1 to x_6 = nill	7.64	52.696
37	x_1 to x_6 = nill	7.93	53.347
38	x_1 to x_6 = nill	8.23	53.988
39	x_1 to x_6 = nill	8.52	54.617
40	x_1 to x_6 = nill	8.82	55.235
41	x_1 to x_6 = nill	9.13	55.842
42	x_1 to x_6 = nill	9.43	56.439

43	x_1 to $x_6 =$ nill	9.74	57.024
44	x_1 to $x_6 =$ nill	10.04	57.599
45	x_1 to $x_6 =$ nill	10.35	58.164
46	x_1 to $x_6 =$ nill	10.66	58.718
47	x_1 to $x_6 =$ nill	10.97	59.263
48	x_1 to $x_6 =$ nill	11.28	59.797
49	x_1 to $x_6 =$ nill	11.60	60.321
50	x_1 to $x_6 =$ nill	11.91	60.836
51	$x_1 = 0.0737$, $x_2 = -0.0563$, $x_3 = 0.0192$, $x_4 = 0.4714$, $x_5 = -0.1788$, $x_6 = -0.1705$	nill	Nill

TABLE 2: Generated from Problem 2

Number of Iteration	Minimizing vectors$X$	Objective function	Gradient
1	x_1 to x_6 =nill	nill	2.449
2	x_1 to x_6 =nill	0.99	1.110
3	x_1 to x_6 =nill	0.99	2.598
4	x_1 to x_6 =nill	1.00	3.647
5	x_1 to x_6 =nill	1.01	3.931
6	x_1 to x_6 =nill	1.02	3.987
7	x_1 to x_6 =nill	1.04	4.060
8	x_1 to x_6 =nill	1.05	4.240
9	x_1 to x_6 =nill	1.06	4.564
10	x_1 to x_6 =nill	1.07	5.050
11	x_1 to x_6 =nill	1.09	5.705
12	x_1 to x_6 =nill	1.10	6.538
13	x_1 to x_6 =nill	1.12	7.551
14	x_1 to x_6 =nill	1.14	8.746
15	x_1 to x_6 =nill	1.17	10.120
16	x_1 to x_6 =nill	1.20	11.665
17	x_1 to x_6 =nill	1.23	13.375
18	x_1 to x_6 =nill	1.27	15.229
19	x_1 to x_6 =nill	1.31	17.217
20	x_1 to x_6 =nill	1.36	19.319
21	x_1 to x_6 =nill	1.41	21.516
22	x_1 to x_6 =nill	1.47	23.788
23	x_1 to x_6 =nill	1.54	26.117
24	x_1 to x_6 =nill	1.61	28.485

25	x_1 to x_6 =nill	1.69	30.874
26	x_1 to x_6 =nill	1.78	33.272
27	x_1 to x_6 =nill	1.87	35.663
28	x_1 to x_6 =nill	1.96	38.038
29	x_1 to x_6 =nill	2.07	40.387
30	x_1 to x_6 =nill	2.17	42.701
31	x_1 to x_6 =nill	2.29	44.974
32	x_1 to x_6 =nill	2.41	47.201
33	x_1 to x_6 =nill	2.53	49.377
34	x_1 to x_6 =nill	2.66	51.501
35	x_1 to x_6 =nill	2.80	53.569
36	x_1 to x_6 =nill	2.94	55.579
37	x_1 to x_6 =nill	3.09	57.532
38	x_1 to x_6 =nill	3.24	59.427
39	x_1 to x_6 =nill	3.39	61.263
40	x_1 to x_6 =nill	3.55	63.042
41	x_1 to x_6 =nill	3.71	64.764
42	x_1 to x_6 =nill	3.88	66.436
43	x_1 to x_6 =nill	4.05	68.041
44	x_1 to x_6 =nill	4.23	69.599
45	x_1 to x_6 =nill	4.41	71.104
46	x_1 to x_6 =nill	4.59	72.559
47	x_1 to x_6 =nill	4.78	73.965
48	x_1 to x_6 =nill	4.97	75.324
49	x_1 to x_6 =nill	5.16	76.636
50	x_1 to x_6 =nill	5.36	77.904
51	$x_1 = -0.2289$ $x_2 = -0.0724$ $x_3 = 0.2464$ $x_4 = -0.0587$ $x_5 = 0.1224$ $x_6 = 0.0549$	5.56	Nill

TABLE 3: Generated from Problem 3

Number of Iteration	Minimizing vectors$X$	Objective Function	Gradient
1	$x_1 = 0.000$ $x_2 = 0.000$ $x_3 = 0.000$ $x_4 = 0.000$ $x_5 = 0.000$ $x_6 = 0.000$	nil	2.449

2	$x_1 = -0.261$ $x_2 = -0.261$ $x_3 = -0.261$ $x_4 = -0.261$ $x_5 = -0.261$ $x_6 = -0.261$	0.22	19.149
3	$x_1 = 0.033$ $x_2 = 0.174$ $x_3 = 0.108$ $x_4 = 0.061$ $x_5 = 0.071$ $x_6 = 0.118$	1.28	
4	$x_1 = -0.013$ $x_2 = 0.124$ $x_3 = 0.070$ $x_4 = 0.045$ $x_5 = 0.068$ $x_6 = 0.127$	1.21	1.978
5	$x_1 = -0.024$ $x_2 = 0.106$ $x_3 = 0.007$ $x_4 = 0.032$ $x_5 = 0.060$ $x_6 = 0.147$	1.16	0.234
6	$x_1 = -0.022$ $x_2 = 0.103$ $x_3 = 0.008$ $x_4 = 0.031$ $x_5 = 0.053$ $x_6 = 0.1152$	1.16	0.006
7	$x_1 = -0.022$ $x_2 = 0.103$ $x_3 = 0.008$ $x_4 = 0.032$ $x_5 = 0.053$ $x_6 = 0.152$	1.16	0.000

TABLE 4: Generated from Problem 4

Number of Iteration	Minimizing vectors X	Objective function	Gradient
1	$x_1 = 0.000$ $x_2 = 0.000$ $x_3 = 0.000$ $x_4 = 0.000$ $x_5 = 0.000$ $x_6 = 0.000$	nil	2.449
2	$x_1 = -0.053$ $x_2 = -0.053$ $x_3 = -0.053$ $x_4 = -0.053$ $x_5 = -0.053$ $x_6 = -0.053$	0.84	4.114
3	$x_1 = -0.071$ $x_2 = -0.321$ $x_3 = -0.201$ $x_4 = -0.163$ $x_5 = -0.153$ $x_6 = -0.162$	0.46	2.970
4	$x_1 = -0.040$ $x_2 = -0.367$ $x_3 = -0.217$ $x_4 = -0.211$ $x_5 = -0.206$ $x_6 = -0.199$	0.38	1.930
5	$x_1 = -0.040$ $x_2 = -0.395$ $x_3 = -0.198$ $x_4 = -0.230$ $x_5 = -0.226$ $x_6 = -0.209$	0.35	0.092
6	$x_1 = -0.040$ $x_2 = -0.395$ $x_3 = -0.198$ $x_4 = -0.228$ $x_5 = -0.228$ $x_6 = -0.209$	0.35	0.019

7	$x_1 = -0.040$ $x_2 = -0.395$ $x_3 = -0.198$ $x_4 = -0.228$ $x_5 = -0.228$ $x_6 = -0.209$	0.35	0.000

4.2 Discussion of Results

The initial nuclear tokens used in problems 1 and 2 to represent the vectors and control operators of the quadratic model were arbitrary. Our results clearly shown that arbitrary composition of nuclear tokens will not guarantee safety. This is evidence from tables 1 and 2 (non convergence) which did not satisfied the properties of the CGM algorithm. See [10].

After restructuring, nuclear tokens were used as the vectors and control operators of the quadratic model to generate problems 3 and 4 and were solved using the CGM algorithm. We were able to get two results that converge (Tables 3 and 4). The convergency satisfied the properties of the CGM algorithm, which shows good results. See [11] and [12].

5.0 Conclusion

The nuclear reactors tokens are the main components that make up the reactors, example of the components are Internal energy, Reactor volume, Molar mass and so on, which produces the rate of heat of the reactor that causes the accidents. See
[13]-[20].

Our results clearly indicate that if the nuclear tokens used for the structured model, are use for the construction of nuclear reactors, then the nuclear safety will be maximize while the disaster will be minimized.

6.0 References

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