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External Fluid Breakthrough Time Characterization of Horizontal Wells in a Layered Reservoir with Letter 'F' Architecture and Edge External Fluid Drive

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Abstract

When external fluid (water, gas or even oil) breaks through a well, its source is seldom known with certainty. Information about the source of an external fluid, no matter the kind, is required by reservoir and completion engineers for correct advising on well geometry and completion strategy to either curb its influx or prevent it. External fluid breakthrough is preceded by fluid influx and may manifest steady state flow in the well if its compressibility is slow. Sustaining production at the prevailing rate will therefore lead to increasing volume of the "unwanted" fluid. Only production at an optimum rate would guarantee clean oil production. In terms of the required drawdown, the optimum rate and critical pressure drawdown are strongly dependent upon the breakthrough time; the largest time fluid production is possible under a constant rate without external fluid breakthrough. For largely permeable interface and layers fluids of close or different properties, simultaneous individual layer production and characterization can be achieved if regional fluid breakthrough times are known.

In this paper, external fluid breakthrough times for horizontal wells completed in each layer of a vertically-stacked layered reservoir with an architecture similar to letter 'F' will be derived based on reservoir pressure distribution. The derivation will consider all the possible edge external fluid influences in the architecture. The interface will be considered separately as crossflow and no-crossflow. Results show that breakthrough times are governed by reservoir layer, wellbore and reservoir or reservoir layers' and fluid properties given a constant production rate history.

Breakthrough time expressions contain only the functions xii(x), xiii(x), vi(z) and ix(z) in the equations to be solved. They are longer for longer wells and are affected by interface permeability. For no-crossflow cases, breakthrough time is not affected directly by layers' thickness, but is delayed where lateral lengths are greater than their thickness.

1.0 Introduction

External fluid breakthrough time is the time a fluid outside the reservoir reaches the wellbore during production of oil or gas. Before breakthrough, the fluid first recharges the reservoir and reduces the reservoir pressure decline. In this case, even though oil/gas production is going on from the reservoir through the wellbore, the observed static bottom hole pressure would remain high. Except the production rate is lowered the external fluid soon reaches the wellbore and is produced afterwords. In otherwords, if the time of breathrough is prolonged, the external fluid may never reach the wellbore. If a reservoir has an external fluid support, reservoir engineers normally try to understand the readiness of such fluid to flow. One tool that is commonly applied to achieve this understanding is the reservoir pressure distribution. The distribution shows the magnitude of the entire fluid energy in a reservoir as can be felt by the well drilled and completed in the reservoir. Reservoir, fluid, wellbore properties and withdrawal/injection rates are variables in the pressure distribution expression in real time. Once the

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dimensionless pressure distribution expression is derived, then information like reservoir anisotropy, geometry, certain rate histories can be determined.

External fluid influx (or breakthrough) could mar oil or gas production economics. Apart from assisting to decide well rates for economic production, external fluid arrival times and patterns are useful in interpreting reservoir geometry or architecture. In infill or further development wells drilling, reservoir geometry is often required for well location for optimum recovery.

In layered reservoirs, where permeability distribution largely varies through the reservoir, one well may be used to produce oil or gas from all the layers or each layer's fluid may be produced separately, if there is crossflow or no-crossflow interface, respectively. If one or more layers experience edge fluid influx, this paper predicts characteristic arrival times of such influx into horizontal well completed in each of the layers, assuming that the layered reservoir has a geometry similar to letter 'F' shown in **Fig. 1**

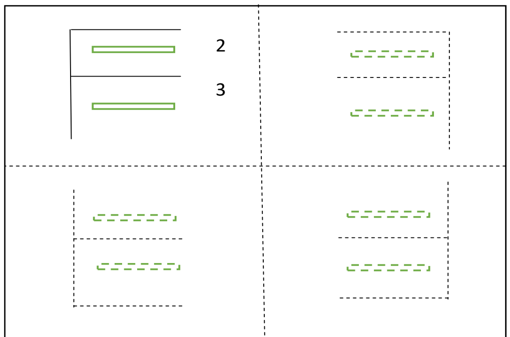
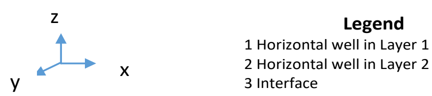


Fig. 1: Vertically-Stacked Two Layered Reservoir with Normal and Inverted Letter 'F' Architecture[7]



Dimensionless pressure expressions describing oil flow in a vertically-stacked layered reservoir of letter 'F' architecture[7] are utilised. Both crossflow and no-crossflow interfaces are considered. Only edge external fluid drive models, where external movement is parallel to reservoir bedding, are derived.

External fluid movement into horizontal wells for single compartment reservoirs have been characterised[1-2]. For layered reservoirs, Refs.[3-6], have characterized external fluid breakthrough time for horizontal wells in layered reservoirs of different architectures. Ref.[7] describes edge external fluid movement in detail. Derivations are for both normal inverted forms of the architecture.

2.0 Derivation of External Fluid Breakthrough Time

According to Ref.[7], the dimensionless pressure for a reservoir or reservoir layer is

$$p_D = 2\pi h_D E\int_0^{t_D} s(x_D,\tau).s(y_D,\tau).s(z_D,\tau)d\tau \qquad (1)$$

During external fluid influx into a reservoir, especially for fluid of low compressibility, there is no change in reservoir pressure with time. This is steady state characteristic represented mathematically as:

$$\frac{\partial p_D}{\partial t_D} \approx 0 \qquad (2)$$

Applying Eq. 2 to Eq. 1 and solving yields

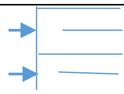
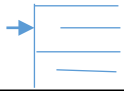
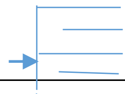
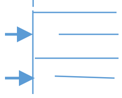
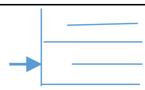
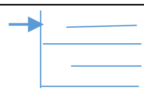
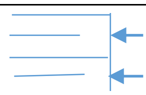
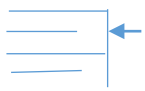
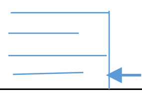
$$s(x_D,t_D).s(y_D,t_D).s(z_D,t_D) = 0 \qquad (3)$$

The t_D at which Eq. 3 holds is the desired edge external fluid breakthrough time, t_{Dbr}. In Eq. 3, the source functions describe the different external boundaries of the reservoir layers, including edge external fluid (constant-pressure), sealing (no-flow) or infinite-acting[7] boundaries as shown in **Table 1**. In this paper, Eq. (3) is solved for t_{Dbr} for all the edge external fluid drive boundaries identified.

3.0 Results and Discussion

Table 2 shows the source functions to be substituted in Eq. 3. Note that both infinite and bounded sources from y-axis have been considered. Each of them is ultimately zero in the process of solution. Similarly, neither infinite-acting nor sealing external boundary would exist upon substitution in Eq. 3. That is, only boundaries with edge external fluid influx and/or interface crossflow would produce source functions into Eq. 3, since only the final flow periods are considered.

Table 1: Compilation of Equations to be solved for Edge External Fluid Breakthrough Time

Model Number	**Model Diagram**	**Equations to be solved**			
		Crossflow		**No-Crossflow**	
		Layer 1	**Layer 2**	**Layer 1**	**Layer 2**
1.		xiii(x)	xii i(x).vi(z)	xiii(x)	xiii(x)
2.		ix(z)	xiii(x).vi(z)	N.A.	xiii(x)
3.		xiii(x)	vi(z)	xiii(x)	N.A.
4.		xiii(x).ix(z)	xiii(x).vi(z)	xiii(x)	xiii(x)
5.		xiii(x).ix(z)	vi(z)	xiii(x)	N.A.
6.		ix(z)	xiii(x).vi(z)	N.A.	xiii(x)
7.		xii(x).ix(z)	xii(x).vi(z)	xii(x)	xii(x)
8.		ix(z)	xii(x).vi(z)	N.A.	xii(x)
9.		xii(x).vi(z)	vi(z)	xii(x)	N.A.

For crossflow layers, sources like xii(x), xiii(x), which are slab sources characterizing edge external fluid influx along the axis of the well length, are common. For the fact that the interface is considered as an internal boundary of the architecture, two fluid types are likely admissible. These fluids are oil or gas, or both. Pressure gradients and fluid ratios can reveal the actual fluid or proportion of fluids migrating across the interface at any flow time. Functions like vi(z), ix(z), model fluid movement across the interface in Layer 2 and Layer 1, respectively.

No-crossflow layers do not contain any plane source from a slab reservoir. That is, the layers breakthrough times are characterized by only infinite slab source from slab reservoirs, i.e., xiii(x) and xii(x).

Therefore, breakthrough times for crossflow layers are affected directly by reservoir length and thickness. As shown in the solution to the demonstration problem, larger reservoir length and thickness give prolong breakthrough time. The reservoir length referred to is the distance of the perforations from the point of edge external fluid contact with the reservoir. Such distances are expected to be closed to infinite-acting boundaries. For a fixed pay thickness, the most effective workover to delay fluid breakthrough would be a shift of the perforation positions from the well heels as far as practicable. Where only pay thickness (Models 3, 5 and 9) determines breakthrough time, early breakthrough problem could be solved by longer well lengths.

Breakthrough times for all forms of the letter ‘F’ architecture are not affected by layers’ pay thickness for all no-crossflow cases. Only length modification along the axis of the well can be effective in effort to delay edge external fluid drive.

4.0 Demonstration Problem

A horizontal well was completed in each layer of a vertically-stacked two-layered reservoir, believed to be of letter ‘F’ architecture. The upper layer is known to be subject to single edge water drive. It is desired to characterise the wells using their breakthrough time, considering the effects of the interface.

5.0 Solution to the Demonstration Problem

The appropriate model for this problem is identified from Table 1 as **Model 2**, reproduced in **Fig 2**. The layered reservoir will be considered as possibly having crossflow and no-crossflow interface separately, since there is no clear evidence of the actual nature of the interface.

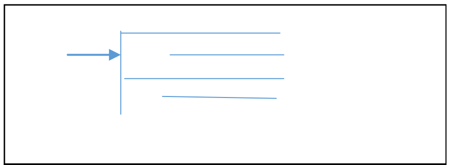


Fig. 2: Model 2; Demonstration Problem

6.0 Crossflow Layers

The layers equations to be solved are derived as follows:

Layer 1

x-axis

The well being at the centre, experiences an infinite slab source from an infinite slab reservoir, with the bottom (heel) sealed and the top (toe) infinite. For the purpose of flow, even the infinite-acting boundary would act as if it were sealed at long flow time occasioned by attenuation of flow streamlines by the bottom sealing boundary. Hence, from Table 2, the appropriate source when the external boundaries are felt (long flow time) is selected as source number s(x,t) = x(x).

y-axis

Along the y-axis, the source is an infinite plane source in an infinite plane reservoir. From Table 2, source number s(y,t) = vii(y) is selected.

z-axis

Layer 1 is bounded at the top by a permeable interface and an infinitely far away bottom boundary. At long flow time therefore, the source is an infinite plane source in an infinite plane reservoir represented by source function number s(z,t) = ix(z).

Therefore, the final equation to be solved according to Eq. (3) is

x(x) vii(y) ix(z) = 0 (4)

Or, substituting the full functions from Table 2, we have

$$\frac{x_f}{x_e}\left[1+\frac{4x_e}{\pi x_f}\sum_{n=1}^{\infty}\exp(-\frac{n^2\pi^2\eta_x t}{x_e^2})\sin\frac{n\pi x_f}{2x_e}\cos\frac{n\pi x_w}{x_e}\cos\frac{n\pi x}{x_e}\right] \cdot \frac{1}{y_e}\left[1+2\sum_{n=1}^{\infty}\exp(-\frac{n^2\pi^2\eta_y t}{y_e^2})\cos\frac{n\pi y_w}{y_e}\cos\frac{n\pi y}{y_e}\right] \cdot$$

$$\frac{2}{h}\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\eta_z t}{4h^2})\cos\frac{(2n+1)\pi z_w}{h}\cos\frac{(2n+1)\pi z}{h}=0 \quad (5)$$

Solving Eq. (5), the only component that will be left because of the infinite summation of an exponential series is

$$\frac{2}{h}\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\eta_z t}{4h^2})\cos\frac{(2n+1)\pi z_w}{h}\cos\frac{(2n+1)\pi z}{h}=0 \quad (6)$$

Or simply

ix(z) = 0 (7)

Solving further, taking n = 1, (number of images at the wellbore) and consider ring only the first two terms of the infinite series, we have

$$t=t_{br}\approx\frac{4h^2}{9\pi^2\eta_z} \quad (8)$$

t_{Dbr} can be obtained by substituting Eq. (8) in Eq. (16).

Layer 2

For Layer 2, using a similar argument as in Layer 1, we have the following equation to solve

xiii(x).vii(y).vi(z) = 0 (9)

Eq. (9) is produced as follows from Table 2 after solving:

$$xiii(x).vi(z)=\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\exp(-\frac{(2n-1)^2\pi^2\eta_x t}{4x_e^2})\cos\frac{(2n-1)\pi x_f}{4x_e}\sin\frac{(2n-1)\pi x_w}{2x_e}\sin\frac{(2n-1)\pi x}{2x_e}.$$

$$\frac{2}{h}\sum_{n=1}^{\infty}\exp(-\frac{(2n-1)^2\pi^2\eta_z t}{4h^2})\sin\frac{(2n-1)\pi z_w}{2h}\sin\frac{(2n-1)\pi z}{2h} \quad . \quad (10)$$

Similarly, solving Eq. (10) taking n = l = 1, yields

$$t=t_{br}\approx\frac{1}{\frac{\pi^2}{4}\left[\frac{\eta_x}{x_e^2}+\frac{\eta_z}{h^2}\right]} \quad (11)$$

7.0 No Crossflow Layers

Because the interface does not affect fluid flow dynamics in the layers, the following equations are to be solved:

Layer 1

No crossflow through the interface and no edge external fluid drive.

Layer 2

xiii(x) = 0 (12)

The solution to Eq. (12) is

$$t = t_{br} \approx \frac{4x_e^2}{\pi^2 \eta_x} \tag{13}$$

8.0 Conclusion

Edge external fluid breakthrough time for horizontal wells in a vertically-stacked two-layered reservoir with letter 'F' architecture are:

1. Shown to contain only the functions xii(x), xiii(x), vi(z) and ix(z) (or a combination) in the equations to be solved.
2. Longer for longer wells.
3. Strongly affected by interlayer interface permeability.
4. Enhanced for reservoir layers where lateral lengths are greater than their thickness (rectangular reservoirs).
5. Not affected directly by layers' thickness for no-crossflow cases.
6. Delayed for reservoir and fluid of low diffusivity constants.

9.0 References

[1] Ozkan, E., and Raghavan, R., "Performance of Horizontal Wells Subject to Bottom Water Drive," SPE Paper 18545, presented at the SPE Eastern Regional Meeting, Charleston, West Virginia, Nov. 2–4, 1988.

[2] Papatzacos, P., Herring, T. U., Martinsen, R., and Skjaeveland, S. M., "Cone Breakthrough Time for Horizontal Wells," SPE Paper 19822, presented at the 64th SPE Annual Conference and Exhibition, San Antonio, TX, Oct.8–11, 1989.

[3] Adewole, E.S. "Theoretical Dimensionless Breakthrough Time Of A Horizontal Well In A Two-Layered Reservoir System With Varying Architecture, Part I: Letter 'B' Architecture, Edge Water Drive Mechanism," *Advanced Material Research*, vol. 367, p. 375-384, 2012.

[4] Adewole, E.S.: "Theoretical Dimensionless Breakthrough Time of A Horizontal Well In A Two-Layered Reservoir System With Varying Architecture Part II: Letter 'B' Architecture, Bottom Water Drive, Mechanism," *Advanced Material Research*, vol. 367, p. 375-384, 2012.

[5] Adewole, E.S. and Olafuyi, O.A.: "Theoretical Dimensionless Breakthrough Time of a Horizontal Well in a Vertically-Stacked Two-Layered Reservoir System with Varying Architecture Part III: Letter 'e' Architecture, Edge Water Drive Mechanism," paper accepted for publication in *J. of Science and Technology, Uniben,* 2013.

[6] Adewole, E.S.: "Theoretical Dimensionless Breakthrough Time of a Horizontal Well in a Vertically-Stacked Two-Layered Reservoir System with Varying Architecture Part IV: Letter 'e' Architecture, Bottom Water Drive Mechanism," paper accepted for publication in *J. of Science and Technology, Uniben,* 2013.

[7] Adewole, E.S.: "Dimensionless Pressure Distribution of a Layered Reservoir with Letter 'F' Architecture Subject to Edge External Fluid Drive," paper submitted for consideration for publication in NAMP, 2015.

Nomenclature

B	Formation volume factor, rbbl/stb
c_t	Total compressibility, 1/psi
E_1	Late time flow boundary effects constant for Layer 1
E_2	Late time flow boundary effects constant for Layer 2
(i)	Axial flow directions x, y, or z
h	Layer pay thickness, ft
k	Average geometric permeability, md
L	Well length, ft
Δp	Pressure drop, psi
q	Flow rate, bbl/day
μ	Reservoir fluid viscosity, cp
ϕ	Porosity, fraction
η	Diffusivity constant, md-psi/cp
s	Source
t	Time, hours

Subscripts

br Breakthrough

D Dimensionless

e External

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}} \tag{14}$$

$$p_D = \frac{kh\Delta p}{141.2q\mu B} \tag{15}$$

$$t_D = \frac{0.000264 \times 4 \times kt}{\phi\mu c_t L^2} \tag{16}$$

$$h_D = \frac{2h}{L}\sqrt{\frac{k}{k_x}} \tag{17}$$

Appendix

Table 2: Source and Green's Functions Required in Equation (3)

S/N	Function Type	Basic Green's Functions	Green's Functions Number
1.	Infinite plane	$\exp\left[-(i-i')^2/4\eta_i t\right]/2\sqrt{\pi\eta_i t}$	i(i)
2.	Infinite slab	$\frac{1}{2}\left[erf\,(x_f/2+(i-i')/2\sqrt{\eta_i t})+erf\,(x_f/2-(i-i')/2\sqrt{\eta_i t})\right]$	ii(i)
Basic Source Functions for Late Flow Periods			
3.	Infinite plane in an infinite slab reservoir with sealed boundaries	$\frac{1}{i_e}\left[1+2\sum_{n=1}^{\infty}\exp(-\frac{n^2\pi^2\eta_i t}{i_e^2})\cos\frac{n\pi i_w}{i_e}\cos\frac{n\pi i}{i_e}\right]$	vii(i)
4.	Infinite plane source in an infinite slab reservoir with constant-pressure boundaries	$\frac{2}{i_e}\sum_{n=1}^{\infty}\exp(-\frac{n^2\pi^2\eta_i t}{i_e^2})\sin\frac{n\pi i_w}{i_e}\sin\frac{n\pi i}{i_e}$	iv(i)
5.	Infinite plane source in an infinite slab reservoir with a sealed boundary at the bottom and a constant-pressure boundary at the top	$\frac{2}{i_e}\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\eta_i t}{4i_e^2})\cos\frac{(2n+1)\pi i_w}{i_e}\cos\frac{(2n+1)\pi i}{i_e}$	ix(i)
6.	Infinite plane source in an infinite slab reservoir with a constant-pressure boundary at the bottom and a sealed boundary at the top	$\frac{2}{i_e}\sum_{n=1}^{\infty}\exp(-\frac{(2n-1)^2\pi^2\eta_i t}{4i_e^2})\sin\frac{(2n-1)\pi i_w}{2i_e}\sin\frac{(2n-1)\pi i}{2i_e}$	vi(i)
7.	Infinite slab source in an infinite slab reservoir with both boundaries sealed	$\frac{x_f}{i_e}\left[1+\frac{4i_e}{\pi x_f}\sum_{n=1}^{\infty}\exp(-\frac{n^2\pi^2\eta_i t}{i_e^2})\sin\frac{n\pi x_f}{2i_e}\cos\frac{n\pi i_w}{i_e}\cos\frac{n\pi i}{i_e}\right]$	x(i)
8.	Infinite slab source in an infinite slab reservoir with constant-pressure boundaries at both ends	$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-\frac{n^2\pi^2\eta_i t}{i_e^2})\sin\frac{n\pi x_f}{2i_e}\sin\frac{n\pi i_w}{i_e}\sin\frac{n\pi i}{i_e}$	viii(i)
9.	Infinite slab source in an infinite slab reservoir with a sealed bottom boundary and a constant-pressure top boundary	$\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n+1}\exp(-\frac{(2n+1)^2\pi^2\eta_i t}{4i_e^2})\sin\frac{(2n+1)\pi i_w}{2i_e}\cos\frac{(2n+1)\pi i}{2i_e}\cos\frac{(2n+1)\pi i}{2i_e}$	xii(i)
10.	Infinite slab source in an infinite slab reservoir with a sealed top boundary and a constant-pressure bottom boundary	$\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\exp(-\frac{(2n-1)^2\pi^2\eta_i t}{4i_e^2})\cos\frac{(2n-1)\pi x_f}{4i_e}\sin\frac{(2n-1)\pi i_w}{2i_e}\sin\frac{(2n-1)\pi i}{2i_e}$	xiii(i)