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Dimensionless Pressure Distribution of a Layered Reservoir with Letter 'F' Architecture Subject to Edge External Fluid Drive

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Abstract

Reservoir architecture may vary widely, giving rise to an architecture akin to Letter "F" in addition to compartmentalization or layering. If such reservoirs are subject to edge external fluid drive, well completion strategies, rate profiling, well re-completion, would require informed knowledge of boundary distribution and boundary types. All these tasks can be accomplished through correct understanding of pressure distributions of the reservoir system.

This paper identifies all the possible edge external fluid combinations that may drive fluid in a reservoir with letter "F" architecture and derives possible real time dimensionless pressure expressions for horizontal wells in the layers of the reservoir.

Both crossflow and no-crossflow interface cases were considered. Appropriate source and Green's functions were selected to build the dimensionless pressure models.

Results obtained show that layered reservoir with letter 'F' architecture presents three (3) external boundary types that may be sealing or constant-pressured, and three (3) mandatory infinite-acting external boundaries that can neither be sealing nor constant-pressured. Hence, dimensionless pressure expressions derived showed generally that at very early flow times dimensionless flow pressures do not exhibit layering for individual layers for both crossflow and no-flow interfaces. However, for crossflow and no-crossflow interfaces, the dimensionless pressure for the top layer is inversely proportional to reservoir layer area at very late flow times or the reservoir length exposed to flow at intermediate or transitional flow periods preceding late flow period, whether individual or extended reservoir cases.

For individual layer, optimum dimensionless pressure drop to guarantee regional flow only shows strong capability for individual layer characterization. Finally, the most important drivers for well performances were identified as the dimensionless well lengths, radii, reservoir or reservoir layer areas, interface permeability and the reservoir or reservoir layer's fluid properties.

1.0 Introduction

A reservoir experiences edge external fluid drive if the external fluid flows into the reservoir through a direction or directions parralel to the reservoir bedding planes. In this case, only wells nearest the source of edge external fluid will manifest earlier water production than wells farther away. Edge fluid movement into a reservoir can be modelled using dimensionless pressure distribution. Furthermore, dimensionless pressure distributions are invaluable in (1) rate profiling (2) well test analysis formulation (3) reservoir simulation validation (4) well engineering design (5) volumetric material balance calculations and (6) reservoir external boundary characterization.

Reservoir layering results when the reservoir possesses more than one permeability value at different points in the same reservoir. It is the largest heterogeneity traceable to reservoir formation. The process of layering may precipitate an

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architecture likeable to letter 'F' in description. Only seismic impressions can reveal such complex structural event in better detail compared to well logs or other subsurface surveys. Several literature[1-6] have described other architecture that may be encountered. This paper provides general expressions for dimensionless pressure distribution in a vertically-stacked two layered reservoir considering only edge external fluid influences and a horizontal well in each layer. With the interface considered as an internal boundary, the expressions will be derived for crossflow and no-crossflow interface cases.
Real time dimensionless pressure distribution expressions have been derived for several reservoir systems[7-9] . Source and Green's functions discussed in Ref. [10], and as compiled by Refs.[4-7] and Ref.[11] will be utilized throughout the derivations.

2.0 Description of Letter 'F' Layered Reservoir Architecture

Fig. 1 shows the normal and all inverted forms of a vertically-stacked reservoir with letter 'F' architecture. The layers of each of the reservoirs are completed with a horizontal well. The architecture with solid lines represents the normal architecture,while the architectures with broken lines are the inverted forms. In both normal and inverted architectures, all the bounding lines represent external boundaries. The dotted crosslike lines act as 'plane mirrors' producing the image architectures. Edge external fluid can occur only along horizontal directions of the external boundaries. The internal line within the external boundaries in each architecture is an interface. An external boundary can be either sealing; i.e., not allowing flow, or constant-pressured; i.e., allow flow through it. Hence, there are three (3) external boundaries that can be sealing or constant-pressured, one (1) at the lower layer (Layer 1) and two (2) at the upper layer (Layer 2). The normal letter 'F' architecture can be visualized as being constituted by reservoirs akin to letter 'C' at the upper layer and an inverted letter 'L' at the lower layer. There is an interface separating the two layers. The interface may be sealing (no-crossflow) or permeable (crossflow). From Fig. 1, the architecture may be subject to two (2) edge external fluid possibilities having one (1) edge at each layer or one (1) edge external fluid possibility each at the upper and lower layer. The interface is considered as an internal boundary. There is one (1) 'infinitely far away' boundary at each of the layers laterally, but only one (1) 'infinitely far away' at the lower layer along the vertical axis.

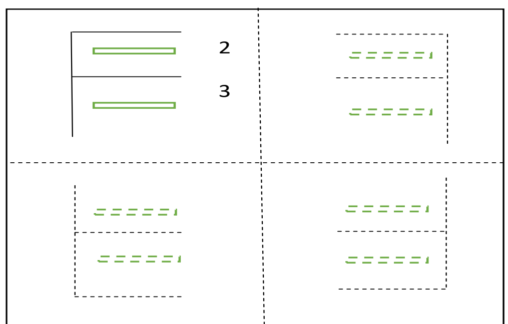
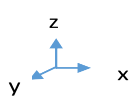


Fig. 1: Vertically-Stacked Two Layered Reservoir with Normal and Inverted Letter



Legend
1 Horizontal well in Layer 1
2 Horizontal well in Layer 2
3 Interface

3.0 Reservoir System Mathematical Description

x-axis

In all cases considered, let the well length coincide with the x-axis and is assumed centrally located. Hence, all source functions for x-axis originate from infinite-acting slab source strength in infinite slab reservoirs at all flow times. Furthermore, particular sources selected will depend on actual nature of external boundary contributing to flow.

y-axis

All the sources coinciding with the well width (y-axis) are considered as infinite plane sources from infinite plane reservoirs, either infinite-acting or sealed. All mirror images of the architecture that produce only vertical stacking of the layers are considered as shown in Fig. 1.

z-axis

Finally, the z-axis has the interface and the top reservoir boundary. If the interface is considered as permeable (crossflow), then it is assumed to produce constant-pressure effect, since both layers pressures and fluid velocities are the same here. For impermeable interface, the z-axis source strength are essentially sealing. However, in both cases of crossflow and no-crossflow, all the z-axes source functions are infinite planes in infinite reservoirs.
All the edge external influx meet the horizontal wells at their heels. All horizontal well toes are in the direction of the 'infinitely far away' lateral boundaries.

4.0 Dimensionless Pressure Distribution Expressure

Generally, using Newman product rule[10-11], the layers dimensionless pressures can be written for no-crossflow layers as

$$p_{Di} = 2\pi h_D \int_0^{t_D} s(x_D,\tau).s(y_D,\tau).s(z_D,\tau)d\tau \qquad (1)$$

and crossflow layers as

$$p_{Di} = 2\pi h_D E \int_0^{t_D} s(x_D,\tau).s(y_D,\tau).s(z_D,\tau)d\tau \qquad (2)$$

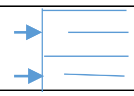
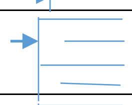
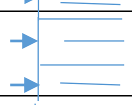
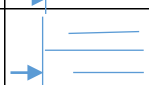
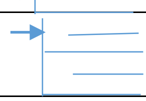
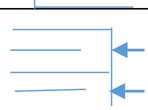
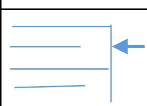
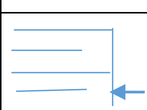
In Eq. 2, E is a weighting factor, which compensates the z-axes sources for the interface effects. The source and Green's functions in both equations are selected from those listed in the Appendix.

5.0 Results and Discussion

Table 1 shows a full compilation of all the possible edge external fluid (constant-pressure) boundaries obtainable for the letter 'F' architecture. There are three (3) normal and six (6) inverted (mirror) versions of the architecture. The mirror image versions are obtained when the mirror is viewed horizontally and vertically. In the normal architecture, the wells lie along the x-axis with their heels adjacent to the vertical z-axis and their toes adjacent to the 'infinitely far away' top boundaries in each layer. The first horizontal image version shows all the well toes now heels and all the well heels now toes. All the architectures obtained simply turned object lower layer top and top layer bottom, but retained the well heels and toes orientation of the parent object wells.

All the dimensionless pressures for cases of crossflow are generally characterized by functions x(x) and ix(z) at late flow times for Layer 1 in all versions of the architecture, depicting edge influx at the heel and crossflow interface, respectively. In Layer 2, vi(z), showing crossflow interface dominate dimensionless pressures. The function x(x) describes boundaries without an edge influx in all crossflow cases. In Layer 2, the function vi(z) dominates flow expressions due to crossflow interface.

Table 1: List of Source and Green's Functions in Equations (1) and (2)

S/N	Model Diagram	s(x,t) . s(x,t) . s(x,t)			
		Crossflow		No-Crossflow	
		Layer 1	Layer 2	Layer 1	Layer 2
1.		ii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).ix(z)	ii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).vi(z)	ii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).vii(z)	ii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).vii(z)
2.		x(x).i(y).i(z) x(x).vii(y).ix(z)	ii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).vi(z)	x(x).i(y).i(z) x(x).vii(y).vii(z)	ii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).vii(z)
3.		ii(x).i(y).i(z) xii(x).i(y).i(z) xiii(x).vii(y).ix(z)	x(x).i(y).i(z) x(x).vii(y).vi(z) ii(x).i(y).i(z)	ii(x).i(y).i(z) xii(x).i(y).i(z) xiii(x).vii(y).vii(z)	x(x).i(y).i(z) x(x).vii(y).vii(z) ii(x).i(y).i(z)
4.		xiii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).ix(z)	xiii(x).vii(y).vi(z) ii(x).i(y).i(z)	xiii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).vii(z)	xiii(x).vii(y).vii(z) ii(x).i(y).i(z)
5.		xiii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).ix(z)	x(x).vii(y).vi(z) ii(x).i(y).i(z) x(x).i(y).i(z)	xiii(x).i(y).i(z) ii(x).i(y).i(z) xiii(x).vii(y).vii(z)	x(x).vii(y).vii(z) ii(x).i(y).i(z) x(x).i(y).i(z)
6.		x(x).vii(y).ix(z) ii(x).i(y).ix(z) x(x).i(y).i(z)	xiii(x).vii(y).vi(z) ii(x).i(y).i(z)	x(x).vii(y).vii(z) ii(x).i(y).vii(z) x(x).i(y).i(z)	xiii(x).vii(y).vii(z) ii(x).i(y).i(z)
7.		xii(x).i(y).i(z) xii(x).vii(y).ix(z) ii(x).i(y).ix(z) ii(x).i(y).i(z)	xii(x).vii(y).vi(z) xii(x).i(y).i(z) ii(x).i(y).vi(z) ii(x).i(y).i(z)	xii(x).i(y).i(z) xii(x).vii(y).vii(z) ii(x).i(y).vii(z) ii(x).i(y).i(z)	xii(x).vii(y).vii(z) xii(x).i(y).i(z) ii(x).i(y).vii(z) ii(x).i(y).i(z)
8.		x(x).i(y).i(z) ii(x).i(y).ix(z) x(x).vii(y).ix(z) ii(x).i(y).i(z)	xii(x).vii(y).vi(z) xii(x).i(y).i(z) ii(x).i(y).vi(z) ii(x).i(y).i(z)	x(x).i(y).i(z) ii(x).i(y).vii(z) x(x).vii(y).vii(z) ii(x).i(y).i(z)	xii(x).vii(y).vii(z) xii(x).i(y).i(z) ii(x).i(y).vii(z) ii(x).i(y).i(z)
9.		xii(x).vii(y).xii(z) xii(x).i(y).i(z) ii(x).i(y).xii(z) ii(x).i(y).i(z)	x(x).vii(y).vi(z) x(x).i(y).i(z) ii(x).i(y).vi(z) ii(x).i(y).i(z)	xii(x).vii(y).vii(z) xii(x).i(y).i(z) ii(x).i(y).vii(z) ii(x).i(y).i(z)	x(x).vii(y).vii(z) x(x).i(y).i(z) ii(x).i(y).vii(z) ii(x).i(y).i(z)

The no-crossflow models show dimensionless pressures characterized by the function vii(z) for all models, because of no flow (sealing, prescribe flux) interface. In all the models considered, possible transition flow periods are suggested. Where there is a pair of sealing or constant pressure external boundary and an infinite-acting boundary, it is assumed that as of the period when the sealing or constant-pressure boundary is felt, the infinite-acting tendency is truncated and the influence of a pseudo-sealing boundary prevails. Purely infinite-acting boundary effects are also suggested for all the models.
Dimensionless pressure drops are generally inversely proportional to the reservoir layer width for Layer 1 in particular and inversely proportional to areal extent for layers not experiencing edge external fluid drive for both crossflow and no-crossflow cases. Except for Models 3, 5 and 9, dimensionless pressures are enhanced if the reservoir or reservoir layer width is less than the reservoir or reservoir layer thickness for both crossflow and no-crossflow cases. Where both lateral extent and pay thickness affect productivity, square horizontal well drainages would reduce the adverse effect of an adjacent lateral boundary. For rectangular reservoir layers, horizontal wells completed along the well length would mitigate the reduction in productivities. Reservoir lateral extent along the length axis of the well do not directly affect pressure drops except where the layer has no edge external fluid drive. All possible transition flow periods between the early radial (infinite-acting) flow periods have been suggested. However, where a transition period involves an external edge drive, such period may terminate further pressure decline especially if the external fluid has low compressibility. Cases where the wells experience all the external boundary influences are really rare if there is more than one external fluid of low compressibility and one of them is felt first. In other words, late time flow periods involving more than one low compressibility fluid are attainable if the external fluid compressibilities are the same and are felt all at the same time.

6.0 Conclusion

All the possible external flow boundaries characterising a vertically-stacked two layered reservoir with letter ‘F’ architecture have been considered in deriving dimensionless pressure obtained with a horizontal well in each layer. Results obtained show that:

(1) There are three (3) normal architecture and six (6) inverted architecture.
(2) In every version of the architecture, there are only three (3) external boundaries that may be sealed or constant-pressured.
(3) Dimensionless pressures for cases of crossflow are characterised at late time by the functions xiii(x) and ix(z) for the bottom layer and vi(z) for the top layer.
(4) Dimensionless pressure for no-crossflow layers are characterised at late time by vii(z).
(5) Dimensionless pressures are inversely proportional to reservoir or reservoir layer width, during transitional flow, and area at late flow time.

7.0 References

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Nomenclature

B	Formation volume factor, rbbl/stb
c_t	Total compressibility, 1/psi
E_1	Late time flow boundary effects constant for Layer 1
E_2	Late time flow boundary effects constant for Layer 2
(i)	Axial flow directions x, y, or z
h	Layer pay thickness, ft
k	Average geometric permeability, md
L	Well length, ft
Δp	Pressure drop, psi
q	Flow rate, bbl/day
μ	Reservoir fluid viscosity, cp
ϕ	Porosity, fraction
s	Source
t	Time, hours

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}}$$

$$p_D = \frac{kh\Delta p}{141.2q\mu B}$$

$$t_D = \frac{0.000264 \times 4 \times kt}{\phi\mu c_t L^2}$$

$$h_D = \frac{2h}{L}\sqrt{\frac{k}{k_x}}$$

Appendix

Table of Source and Green's Functions[9-11]

S/N	Function Type	Basic Green's Functions	Green's Functions Number
1.	Infinite plane	$\exp\left[-(i-i')^2/4\eta_i t\right]/2\sqrt{\pi\eta_i t}$	i(i)
2.	Infinite slab	$\frac{1}{2}\left[erf(x_f/2+(i-i')/2\sqrt{\eta_i t})+erf(x_f/2-(i-i')/2\sqrt{\eta_i t})\right]$	ii(i)
Basic Source Functions for Late Flow Periods			
3.	Infinite plane in an infinite slab reservoir with sealed boundaries	$\frac{1}{i_e}\left[1+2\sum_{n=1}^{\infty}\exp(-\frac{n^2\pi^2\eta_i t}{i_e^2})\cos\frac{n\pi i_w}{i_e}\cos\frac{n\pi i}{i_e}\right]$	vii(i)
4.	Infinite plane source in an infinite slab reservoir with constant-pressure boundaries	$\frac{2}{i_e}\sum_{n=1}^{\infty}\exp(-\frac{n^2\pi^2\eta_i t}{i_e^2})\sin\frac{n\pi i_w}{i_e}\sin\frac{n\pi i}{i_e}$	iv(i)
5.	Infinite plane source in an infinite slab reservoir with a sealed boundary at the bottom and a constant-pressure boundary at the top	$\frac{2}{i_e}\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\eta_i t}{4i_e^2})\cos\frac{(2n+1)\pi i_w}{i_e}\cos\frac{(2n+1)\pi i}{i_e}$	ix(i)
6.	Infinite plane source in an infinite slab reservoir with a constant-pressure boundary at the bottom and a sealed boundary at the top	$\frac{2}{i_e}\sum_{n=1}^{\infty}\exp(-\frac{(2n-1)^2\pi^2\eta_i t}{4i_e^2})\sin\frac{(2n-1)\pi i_w}{2i_e}\sin\frac{(2n-1)\pi i}{2i_e}$	vi(i)
7.	Infinite slab source in an infinite slab reservoir with both boundaries sealed	$\frac{x_f}{i_e}\left[1+\frac{4i_e}{\pi x_f}\sum_{n=1}^{\infty}\exp(-\frac{n^2\pi^2\eta_i t}{i_e^2})\sin\frac{n\pi x_f}{2i_e}\cos\frac{n\pi i_w}{i_e}\cos\frac{n\pi i}{i_e}\right]$	x(i)
8.	Infinite slab source in an infinite slab reservoir with constant-pressure boundaries at both ends	$\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-\frac{n^2\pi^2\eta_i t}{i_e^2})\sin\frac{n\pi x_f}{2i_e}\sin\frac{n\pi i_w}{i_e}\sin\frac{n\pi i}{i_e}$	viii(i)
9.	Infinite slab source in an infinite slab reservoir with a sealed bottom boundary and a constant-pressure top boundary	$\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n+1}\exp(-\frac{(2n+1)^2\pi^2\eta_i t}{4i_e^2})\sin\frac{(2n+1)\pi i_w}{2i_e}\cos\frac{(2n+1)\pi i}{2i_e}\cos\frac{(2n+1)\pi i}{2i_e}$	xii(i)
10.	Infinite slab source in an infinite slab reservoir with a sealed top boundary and a constant-pressure bottom boundary	$\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\exp(-\frac{(2n-1)^2\pi^2\eta_i t}{4i_e^2})\cos\frac{(2n-1)\pi x_f}{4i_e}\sin\frac{(2n-1)\pi i_w}{2i_e}\sin\frac{(2n-1)\pi i}{2i_e}$	xiii(i)