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USING MATRIX OF DEMAND TO DETERMINE PATRONAGE IN HOTELS

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Abstract

The number of rooms in demand in a hotel and the period(s) they are in demand form a matrix we refer to as demand matrix. The number of zero entries in the demand matrix shows the level of patronage the hotel receives from its customers in a period. We use the demand matrix to compare the level of patronage between four hotels with the same subunits.

Keywords: hotel, rooms, demand, patronage, entries.

Introduction.

It is the desire of hotel managers to maintain high level of patronage from its customers. Hotels will go out of their way to satisfy a customer so he/she can always come back. The total number of rooms in a hotel is fixed and subdivided into units [1, 2 , 3]. Some common subunits in Nigeria include; single rooms, double rooms, standard rooms, kings rooms, queens rooms, suites, presidential etc. also, the daily demand for rooms in the hotel cut across all the subunits of the hotel. A brief description of some subunits as given [4] is given below

k_1 => Single Room: A room with one or more beds that is allocated to one person. The size of the room or area measures $37m^2$ to $45m^2$.

k_2 => Double Room A room with one or more beds that is allocated to two people. The size of the room or area measures $40m^2$ to $45m^2$.

k_3 => Triple Room: A room with three twin beds or one double bed and one twin bed that can accommodate three people. The measurement is mostly $45m^2$ to $65m^2$.

k_4 => Quad Room: A room that can accommodate four people and measures $70m^2$ to $85m^2$.

k_5 => Queen Room: A room that can accommodate one or more people with a queen-sized bed and measures $32m^2$ to $50m^2$.

k_6 => King Room: A room that can accommodate one or more people with a king-sized bed and measures $32m^2$ to $50m^2$.

k_7 => Double-double Room: A room that can accommodate two to four people with two double beds and measures $50m^2$ to $70m^2$.

k_8 => Suite/ Executive Suite: One or more bedrooms are linked by a parlour or living room and measures $70m^2$ to $100m^2$.

k_9 => Mini Suite/ Junior Suite: This is a single room that has a bed and sitting area. The sleeping area is sometimes in a bedroom separated from the parlour or living room and measures $60m^2$ to $80m^2$.

k_{10} =>Presidential Suite: This is the most expensive room a hotel has to offer. Usually in a hotel, only one presidential suite is available. A presidential suite, similar to the normal suites, always has one or more bedrooms and a living area, with an emphasis on magnificent in-room decorating, high-quality facilities and supplies, and custom-tailored services. This measures $80m^2$ to $350m^2$.

Description and Derivation of the Demand Matrix.

The demand matrix tells us the number of rooms from each subunits that is in demand and the number of period(s) the room(s) will be in demand. If the total number of rooms in a hotel is K then $K = k_1 + k_2 + k_3 + \quad . \quad . \quad . + k_n$. If 10 rooms are in demand from subunit k_1, such that

2 are in demand for 4 days
1 is in demand for 2 day

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3 are in demand for 3 days
4 are in demand for 1 day
Then we have

$$t_1a_{11}x_1 + t_2a_{12}x_1 + t_3a_{13}x_1 + t_4a_{14}x_1$$

Where t_i is the number of period(s) in demand

a_{11} is the number of rooms from subunit k_1 in demand for one period.

a_{12} is the number of rooms from subunit k_1 in demand for two period.

a_{13} is the number of rooms from subunit k_1 in demand for three period.

a_{14} is the number of rooms from subunit k_1 in demand for four period.

x_1 is the price for a room from subunit k_1.

From the example $a_{11} = 4,\ a_{12} = 1, a_{13} = 3, a_{14} = 2$. If we extend the example to subunits k_2, k_3, k_4 we obtain a system of linear equations.

$$t_1a_{11}x_1 + t_2a_{12}x_1 + t_3a_{13}x_1 + t_4a_{14}x_4$$
$$t_1a_{21}x_2 + t_2a_{22}x_2 + t_3a_{23}x_2 + t_4a_{24}x_2$$
$$t_1a_{31}x_3 + t_2a_{32}x_3 + t_3a_{33}x_3 + t_4a_{34}x_3$$
$$t_1a_{41}x_4 + t_2a_{42}x_4 + t_3a_{43}x_4 + t_4a_{44}x_3$$

In matrix form, the rooms in demand will give

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

Then extending to all subunits in the hotel

$$t_1a_{11}x_1 + t_2a_{12}x_1 + t_3a_{13}x_1 + t_4a_{14}x_4 + \ldots$$
$$t_1a_{21}x_2 + t_2a_{22}x_2 + t_3a_{23}x_2 + t_4a_{24}x_2 + \ldots$$
$$t_1a_{31}x_3 + t_2a_{32}x_3 + t_3a_{33}x_3 + t_4a_{34}x_3 + \ldots$$
$$t_1a_{41}x_4 + t_2a_{42}x_4 + t_3a_{43}x_4 + t_4a_{44}x_3 + \ldots$$
$$\begin{matrix} . & . & . & . \end{matrix}$$
$$\begin{matrix} . & . & . & . \end{matrix}$$
$$\begin{matrix} . & . & . & . \end{matrix}$$
$$t_1a_{n1}x_n + t_2a_{n2}x_n + t_3a_{n3}x_n + t_4a_{n4}x_n + \ldots$$

Which will yield the matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14}\ldots; \\ a_{21} & a_{22} & a_{23} & a_{24}\ldots \\ a_{31} & a_{32} & a_{33} & a_{34}\ldots \\ a_{41} & a_{42} & a_{43} & a_{44}\ldots \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & a_{n4}\ldots \end{pmatrix}$$

The dimension of the matrix depend on the room(s) with the highest period of demand. In the example above, the highest period of demand is 4 days, so the demand matrix is $n \times 4$. If it is 5days then the dimension of the demand matrix will be $n \times 5$.

The total revenue function for a non regular fixed lifetime inventory like the hotel was obtained by [5] and is given as

$$\text{Revenue function} = \sum_{i,j=1}^{n} t_i a_{i,j} x_i - v\int_k^{\infty}(d-k)f(d)dd - \theta\int_0^k(k-d)f(d)dd + P \quad (1)$$

Our interest is to use the demand matrix to compare the level of patronage between hotels. We use the data in [5] for four hotels A, B, C and D. Tables 1, 2, 3, and 4. Show the number of rooms, prices and subunits in the hotels

Table 1: Rooms/ prices in hotel A.

Room type	Number of rooms	Price per room (₦)
Single room	10	4000
Double room	10	8000
Queen room	8	16000
King room	12	16000
Presidential	1	28000

Table 2: Rooms/ prices in hotel B.

Room type	Number of rooms	Price per room (₦)
Single room	15	4000
Double room	10	9000
Queen room	10	15000
King room	10	15000
Suites	4	20000
Presidential	1	30000

Table3: Rooms/ prices in hotel C.

Room type	Number of rooms	Price per room (₦)
Single room	15	4000
Double room	10	7500
Queen room	10	15000
King room	15	15000
Presidential	1	32000

Table 4.: Rooms/ prices in hotel D.

Room type	Number of rooms	Price per room (₦)
Single room	15	4000
Double room	10	9000
Queen room	10	15000
King room	10	15000
Suites	4	20000
Presidential	1	30000

The subunits in hotels A and C are the same and can be compared while the subunits in B and D are the same.
The record obtained from the hotels for a week was analysed with the model in (1) and the result is shown in Tables 5, 6, 7 and 8.

Table 5: Number of rooms in demand, days in demand and revenue from hotel A.

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
1	Single room	10	4000	8 | 2(1) 4(2) 2(3)	2	64000	144000
	Double room	10	8000	2(1)	8	16000	
	Queen room	8	16000	2(2)	6	64000	
	King room	12	16000	0	12	0	
	Presidential	1	28000	0	1	0	
2	Single room	10	4000	4(1) 2(2) ~~2(2)~~	2	16000	56000
	Double room	10	8000	3(1)	7	24000	
	Queen room	8	16000	2(1) ~~0~~	6	0	
	King room	12	16000	1(1)	11	16000	
	Presidential	1	28000	0	1	0	
3	Single room	10	4000	2(1) 2(1) ~~6(1)~~	0	24000	120000
	Double room	10	8000	4(1)	6	32000	
	Queen room	8	16000	1(1)	7	16000	
	King room	12	16000	3(1)	9	48000	
	Presidential	1	28000	0	1	0	

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
4	Single room	10	4000	10 → 5(1), 4(2), 1(3)	0	64000	176000
	Double room	10	8000	6(1)	4	48000	
	Queen room	8	16000	4(1)	4	64000	
	King room	12	16000	0	12	0	
	Presidential	1	28000	0	1	0	
5	Single room	10	4000	4(1), 1(2), 5(1)	0	20000	124000
	Double room	10	8000	7(1)	3	56000	
	Queen room	8	16000	0	8	0	
	King room	12	16000	3(1)	9	48000	
	Presidential	1	28000	0	1	0	
6	Single room	10	4000	1(1), 6(1)	3	24000	64000
	Double room	10	8000	5(1)	5	40000	
	Queen room	8	16000	0	8	0	
	King room	12	16000	0	12	0	
	Presidential	1	28000	0	1	0	
7	Single room	10	4000	8(1)	2	32000	160000
	Double room	10	8000	6(1)	4	48000	
	Queen room	8	16000	2(1)	6	32000	
	King room	12	16000	3(1)	9	48000	
	Presidential	1	28000	0	1	0	

Table 6: Number of rooms in demand, days in demand and revenue from hotel B.

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
1	Single room	15	4000	12 → 4(1), 4(2), 4(3)	3	96000	216000
	Double room	10	9000	5(1)	5	45000	
	Queen room	10	15000	2(1)	8	30000	
	King room	10	15000	3(1)	7	45000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
2	Single room	15	4000	4(1), 4(2), 5(1)	2	20000	230000
	Double room	10	9000	6 → 2(3), 4(1)	4	90000	
	Queen room	10	15000	4(1)	6	60000	
	King room	10	15000	4(1)	6	60000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
3	Single room	15	4000	` 4(1), 8(1)	3	32000	221000
	Double room	10	9000	2(2), 6(1)	2	54000	
	Queen room	10	15000	4(1)	6	60000	
	King room	10	15000	5(1)	5	75000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
4	Single room	15	4000	10(1)	5	40000	115000
	Double room	10	9000	2(1) 5(1)	3	45000	
	Queen room	10	15000	2(1)	8	30000	
	King room	10	15000	0	10	0	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
5	Single room	15	4000	12(1)	3	48000	189000
	Double room	10	9000	4(1)	6	36000	
	Queen room	10	15000	3(1)	7	45000	
	King room	10	15000	4(1)	6	60000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
6	Single room	15	4000	10(1)	5	40000	250000
	Double room	10	9000	5(1)	5	45000	
	Queen room	10	15000	6(1)	4	90000	
	King room	10	15000	5(1)	5	75000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
7	Single room	15	4000	11(1)	4	44000	209000
	Double room	10	9000	0	10	0	
	Queen room	10	15000	4(1)	6	60000	
	King room	10	15000	7(1)	3	105000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	

Table 7: Number of rooms in demand, days in demand and revenue from hotel C.

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
1	Single room	15	4000	2(1) 5 1(2) 2(3)	10	40000	90000
	Double room	10	7500	1(2)	9	15000	
	Queen room	10	15000	2(1)	8	20000	
	King room	15	15000	1(1)	14	15000	
	Presidential	1	32000	0	1	0	
2	Single room	15	4000	1(1) 2(2) 8(1)	4	32000	114500
	Double room	10	7500	3(1)	7	22500	
	Queen room	10	15000	2(2)	8	60000	
	King room	15	15000	0	15	0	
	Presidential	1	32000	0	1	0	
3	Single room	15	4000	2(1) 4(1) 9 2(2) 3(3)	4	68000	173000
	Double room	10	7500	2(1)	8	15000	
	Queen room	10	15000	0	10	0	
	King room	15	15000	3(2)	12	90000	
	Presidential	1	32000	0	1	0	

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
4	Single room	15	4000	2(1) 3(2) 5(1)	5	20000	125000
	Double room	10	7500	2(2)	8	30000	
	Queen room	10	15000	1(1)	9	15000	
	King room	15	15000	2(2)	13	60000	
	Presidential	1	32000	0	1	0	
5	Single room	15	4000	3(1) 7(1)	5	28000	955000
	Double room	10	7500	3(1)	7	22500	
	Queen room	10	15000	2(1)	8	30000	
	King room	15	15000	1(1)	14	15000	
	Presidential	1	32000	0	1	0	
6	Single room	15	4000	8(1)	7	32000	152000
	Double room	10	7500	3(2)	7	45000	
	Queen room	10	15000	2(2)	8	60000	
	King room	15	15000	1(1)	14	15000	
	Presidential	1	32000	0	1	0	
7	Single room	15	4000	10(1)	5	40000	167500
	Double room	10	7500	3(1)	7	22500	
	Queen room	10	15000	3(1)	7	45000	
	King room	15	15000	2(2)	13	60000	
	Presidential	1	32000	0	1	0	

Table 8: Number of rooms in demand, days in demand and revenue from hotel D.

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
1	Single room	15	4000	4(1) 10 3(2) 3(3)	5	76000	223000
	Double room	10	9000	2 1(1) 1(2)	8	27000	
	Queen room	10	15000	0	10	0	
	King room	10	15000	4 2(1) 2(3)	6	120000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
2	Single room	15	4000	3(1) 3(2) 5(1)	4	20000	233000
	Double room	10	9000	1(1) 6(2)	3	108000	
	Queen room	10	15000	1(2)	9	30000	
	King room	10	15000	2(2) 5(1)	3	75000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
3	Single room	15	4000	` 3(1) 7(1)	5	28000	97000
	Double room	10	9000	6(1) 1(1)	3	9000	
	Queen room	10	15000	1(1) 2(1)	7	30000	
	King room	10	15000	2(1) 2(1)	6	30000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
4	Single room	15	4000	4 2(2) 2(3)	11	40000	145000
	Double room	10	9000	3 1(1) 2(2)	7	45000	
	Queen room	10	15000	2(1)	8	30000	
	King room	10	15000	2(1)	8	30000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
5	Single room	15	4000	2(1) 2(2) 5(1)	6	20000	230000
	Double room	10	9000	2(1) 0	8	0	
	Queen room	10	15000	6(1)	4	90000	
	King room	10	15000	4(2)	6	120000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
6	Single room	15	4000	2(1) 7(1)	6	28000	148000
	Double room	10	9000	0	10	0	
	Queen room	10	15000	3(2)	7	90000	
	King room	10	15000	4(1) 1(2)	5	30000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
7	Single room	15	4000	8(1)	7	32000	131000
	Double room	10	9000	1(1)	9	9000	
	Queen room	10	15000	3(1) 2(1)	5	30000	
	King room	10	15000	1(1) 4(1)	5	60000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	

Next we construct the demand matrix for hotels A and C for the first days and compare their level of patronage.

Table 9: zero entries in hotels A and C.

Day	Hotel A	Hotel C
1	$\begin{pmatrix} 2 & 4 & 2 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 10zero entries	$\begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 9zero entries
2	$\begin{pmatrix} 4 & 2 & 0 \\ 3 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 10zero entries	$\begin{pmatrix} 9 & 2 & 0 \\ 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 11zero entries
3	$\begin{pmatrix} 10 & 0 & 0 \\ 4 & 0 & 0 \\ 1 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 12zero entries	$\begin{pmatrix} 6 & 2 & 3 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 10zero entries
Total zeros	32 zero entries	30 zero entries

From Table 9 there are 32 zero entries in hotel A for the first three days and 30 zero entries for hotel C for the first three days, therefore hotel C has more patronage in the period under review.

Next we construct the demand matrix for hotels B and D.

Table 10: zero entries in hotels B and D.

Day	Hotel B	Hotel D
1	$\begin{pmatrix} 4 & 4 & 4 \\ 5 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 12zero entries	$\begin{pmatrix} 4 & 3 & 3 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 11zero entries
2	$\begin{pmatrix} 9 & 4 & 0 \\ 4 & 0 & 2 \\ 4 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 12zero entries	$\begin{pmatrix} 8 & 3 & 0 \\ 1 & 6 & 0 \\ 0 & 1 & 0 \\ 5 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 11zero entries
3	$\begin{pmatrix} 12 & 0 & 0 \\ 6 & 2 & 0 \\ 4 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 13zero entries	$\begin{pmatrix} 10 & 0 & 0 \\ 7 & 0 & 0 \\ 3 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 14zero entries
Total zero	37 zero	36 zero

From Table 10, hotel B has 37 zero entries after three days and hotel D has 36 zero entries after three days, hence hotel D had more patronage in the period under review.

Conclusion

The demand matrix is used by hotel managers as a tool to determine their level of patronage or customers inflow. It is the desire of hotel managers that the entries of the demand matrix should be non-zero entries, as a zero matrix or almost zero matrix will indicate no patronage or low patronage.

Also, the demand matrix can be used to compare the level of patronage (demand) between two hotels. The hotel with the most zero entries in its matrix of demand has lower patronage (demand) when compared with the hotel with the least zero entries in its matrix of demand.

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