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ON EXTENSION OF OPTIMAL REPLACEMENT TIME OF A SERIES SYSTEM

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Abstract

Age replacement policy is optimal among all reasonable replacement policies. The optimal replacement time of a series configuration is shorter when compared to other configurations. As the series system is having the shortest optimal replacement time among other multi-component system, this paper looks for reasonable ways to extend the optimal replacement time of a multi-component systems. The paper considered six series configuration, such that the system is subjected to two types of failures, which are type I and II failures. This paper constructed age replacement model based on standard age replacement policy (SARP) for a series system. Furthermore, some two additional age replacement model are also constructed under some proposed policies, which are policy A and policy B. Finally, a numerical example is given for simple illustration of the models constructed under SARP, policy A and policy B.

Keywords**:** Failure, Optimal, Policy, Repair, Series.

1. Introduction

The importance of reliability has been increasing greatly with the innovation of recent technology. The theory has been actually applied to industrial, electronics, computer, information and communication engineering. Many researchers have investigated statistically and stochastically complex phenomena of some real systems, so as to improve their reliability. Most systems deteriorate and subsequently fail due to age and usage. Such failures have negative effect on revenue, production of defective items and causes delay in customer services. To reduce the incidences of system failures, management of organizations is always interested with implementing an appropriate preventive replacement policy for normal system operation. For these reasons, many researchers developed several optimal replacement policies for reducing unnecessary high operating costs.

In [1], a discounted replacement model for a unit subjected to two types of failures (type I and type II failures) was presented, such that, if the failure is of type 1, the system is minimally repaired, while type II is an unrepairable failure. Furthermore, for a system subjected to two types of failures, [2] considered a system which suffers one of two types of failures, such that the system is replaced at a planned time T, at a random working time, or at the first type-II failure, whichever occurs first. In [3], an optimal replacement policy for degenerative system under two types of failures was discussed. In [4], an improved algorithm for obtaining optimal repair/replacement policy for a system with general repairs was introduced. In [5], some properties of the standard age replacement model was explored and discussed. In [6], optimal replacement policy for a repairable system with multiple vacation and imperfect coverage was studied. In [7], a discrete replacement cost model is constructed for a unit, because sometimes a unit cannot be replaced at exact optimum replacement time. In [8], a replacement cost model for a system with vital and non-vital parts is constructed. In [9], integrated bivariate replacement model with warranty was studied. In [10], an age replacement model involving minimal repair was developed, such that, the cost of

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minimal repair and the cost of unplanned replacement depends on time. In [11], a cold standby repairable system with two different components and one repairman who can take multiple vacations was presented. In [12], two parallel units in which both units operate simultaneously was considered, such that, the system is subjected to two types of failures. In [13], the replacement cost model of [1] was modified by introducing random working time Y. In [14], a replacement schedule for non-repairable safety-related systems with multiple components was studied. In [15], continuous scheduled and discrete scheduled replacement times was compared.
This research paper come up with some replacement cost models for a series system under SARP, policy A and policy B. The aim of this paper is to investigate which among the three replacement policies is better for a series system. The scope of this research covers the age replacement model with minimal repair. This paper is organized in five sections. The present section described the introductory part. Section 2 contained the notations, description of the system, methodology and the proposed replacement cost models. Section 3 presents the proposed replacement model. Section 3 presents a numerical example and the results obtained. Section 4 contains the discussion of the results obtained. Finally, section 5 presents the conclusion.

2. Formulation of the Proposed Replacement Cost Models

This section presents a proposed replacement cost model for a series system under standard age replacement policy (SARP) based on some assumptions. Furthermore, this section also presents another two replacement cost models under some proposed replacement policies, which are policy A and policy B.

2.1 Some Basic Notations

1. $r_i(t)$: Type I failure rate of component A_i, for $i = 1,2,3,4,5,6$.
2. $r_i^*(t)$: Type II failure rate of component A_i, for $i = 1,2,3,4,5,6$.
3. $R_i^*(t)$: Reliability function of type II failure for component A_i, for $i = 1,2,3,45,6$.
4. $SARP$: Standard age replacement policy
5. $R_S^*(t)$: Reliability function of type II failure of the system
6. $CS(T)$: Expected cost rate of the system under SARP.
7. $CA(T)$: Expected cost rate of the system under policy A.
8. $CB(T)$: Expected cost rate of the system under policy B.
9. T^*: Optimal replacement time of the system under SARP.
10. T_A^*: Optimal replacement time of the system under policy A.
11. T_B^*: Optimal replacement time of the system under policy B.
12. C_{ir}: Cost of unplanned replacement of failed A_i due to type II failure, for $i = 1, 2, 3, 4, 5, 6$.
13. C_{im}: Cost of minimal repair of failed A_i due to type II failure, for $i = 1, 2, 3, 4, 5, 6$.
14. C_{sp}: Cost of planned replacement of the system at planned replacement time T.
15. C_{sr}: Cost of un-planned replacement of the system due to type II failure.

2.2 System Description

Consider six components A_1, A_2, A_3, A_4, A_5 and A_6, arranged in series configuration to form a system, such that all the six components are subjected to two independent failures, which are type I and type II failures. Type I failure is a degraded failure mode, which occurs due to time and usage. While type II failure is an unrepairable failure, for which a unit fails suddenly and completely. Since the two failures (type I and II) are independent, therefore the failure rates for both type I and II failures are also independent. As mentioned above that each component is subjected to type I and II failures (which are independent), then the system is also subjected to type I and II failures. Therefore, if any of the four components fails due to type I failure, then the system fails due to type I failure, while if any of the four components fails due to type II failure, then the system fails due to type II failure. In this regard, if system fails due to type I failure, the system is minimally repaired and allow the system to continue operating from where it stopped. While if the system fails due to type II failure (non- repairable failure), the whole system is replace completely with new one. Figure 1 below is the diagram of the system.

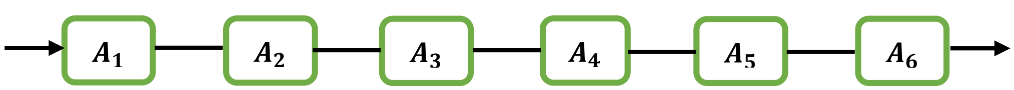

Figure 1. Structure of the series system

2.3 Assumptions of the System Under SARP

1. If the system fails due to type I failure, then the system is minimally repaired.
2. If the system fails due to type II failure, then the system is completely replaced with new one.
3. The replacement and repair time of the failed system is negligible.

4.

5. Both the two failures for the six components arrives according to a non-homogeneous Poisson process.
6. Rate of type II failure follows the order: $r_1^*(t) > r_3^*(t) > r_5^*(t) > r_2^*(t) > r_4^*(t) > r_6^*(t)$.
7. Rate of type I failure follows the order: $r_1(t) > r_3(t) > r_5(t) > r_2(t) > r_4(t) > r_6(t)$.
8. The system is replaced at a planned time $T(T > 0)$ after its installation or at type II failure, whichever arrives first.
9. The cost of planned replacement of the system is less than the cost of un-planned replacement.
10. All costs are positive numbers.

2.4 Replacement Cost Model of the System Under SARP:

Under SARP, the probability that system will be replaced at time T before type II failure occurs is

$$R^*(T) = R_1^*(T)R_2^*(T)R_3^*(T)R_4^*(T)R_5^*(T)R_6^*(T). \quad (1)$$

The cost rate for system based on SARP is

$$C(T) = \frac{C_{sr}(1-R^*(T)) + C_{sp}R^*(T) + \int_0^T K(t)R^*(t)dt}{\int_0^T R_s^*(t)dt}, \quad (2)$$

where

$$K(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t). \quad (3)$$

2.5 Replacement Cost Model of the System Under Policy A:

Policy A: under policy A, the system is replace completely with new one when any of the components A_1, A_3 or A_5 fails due to type II failure. But when any of the components A_2, A_4 or A_6 fails due to type II failure, the failed component is replace completely with new one and allow the system to continue operating from where it stopped. One should not forget that when any among the six components fails due to type I failure, the failed component (due to type I failure) undergo only minimal repair and allow the system to continue operating from where it stopped .

Under policy A, the probability that the system will be replaced at time T before type II failure occurs is

$$R_A^*(T) = R_1^*(T)R_3^*(T)R_5^*(T). \quad (4)$$

The cost rate for system under policy A is

$$CA(T) = \frac{C_{sr}\left(1-R_A^*(T)\right) + C_{sp}R_A^*(T) + \int_0^T K_A(t)R_A^*(t)dt}{\int_0^T R_A^*(t)dt}, \quad (5)$$

where

$$K_A(t) = C_{2r}r_2^*(t) + C_{4r}r_4^*(t) + C_{6r}r_6^*(t) + C_{2m}r_2(t) + C_{4m}r_4(t) + C_{6m}r_6(t) + C_{1m}r_1(t) + C_{3m}r_3(t) + C_{5m}r_5(t). \quad (6)$$

2.6 Replacement Cost Model of the System Under Policy B:

Policy B: under policy B, the system is replace completely with new one when any of the components A_2, A_4 or A_6 fails due to type II failure. But when any of the components A_1, A_3 or A_5 fails due to type II failure, the failed component is replace completely with new one and allow the system to continue operating from where it stopped. One should not forget that when any among the six components fails due to type I failure, the failed component (due to type I failure) undergo only minimal repair and allow the system to continue operating from where it stopped .

Under policy B, the probability that the system will be replaced at time T before type II failure occurs is:

$$R_B^*(T) = R_2^*(T)R_4^*(T)R_6^*(T). \quad (7)$$

The cost rate for system under policy B is

$$CB(T) = \frac{C_{sr}\left(1-R_B^*(T)\right) + C_{sp}R_B^*(T) + \int_0^T K_B(t)R_B^*(t)dt}{\int_0^T R_B^*(t)dt}, \quad (8)$$

where

$$K_B(t) = C_{1r}r_1^*(t) + C_{3r}r_3^*(t) + C_{5r}r_5^*(t) + C_{1m}r_1(t) + C_{3m}r_3(t) + C_{5m}r_5(t) + C_{2m}r_2(t) + C_{4m}r_4(t) + C_{6m}r_6(t). \quad (9)$$

3. Numerical Example

Let the failure time of type I failure for the six components follows Weibull distribution:

$$r_i(t) = \lambda_i \propto_i t^{\propto_i - 1}, \text{ for } i = 1, 2, 3, 4, 5, 6, \quad (10)$$

where $\propto_i > 1$ and $t \geq 0$

Also, let the failure time of type II failure for the six components follows Weibull distribution:

$$r_i^*(t) = \lambda_i^* \propto_i^* t^{\propto_i^* - 1}, \text{ for } i = 1, 2, 3, 4, 5, 6, \quad (11)$$

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where $\propto_i > 1$and $t \geq 0$.

Let the set of parameters and cost of repair/replacement be used throughout this particular example:

1. $\propto_1 = 4$, $\propto_2 = 2$, $\propto_3 = 3$, $\propto_4 = 3$, $\propto_5 = 4$ and $\propto_6 = 2$.
2. $\lambda_1 = 0.03$, $\lambda_2 = 0.03$, $\lambda_3 = 0.03$, $\lambda_4 = 0.001$, $\lambda_5 = 0.001$ and $\lambda_6 = 0.001$.
3. $\propto_1^* = 4$, $\propto_2^* = 3.5$, $\propto_3^* = 4$, $\propto_4^* = 3.5$, $\propto_5^* = 4$, and $\propto_6^* = 3.5$.
4. $\lambda_1^* = 0.00033$,$\lambda_2^* = 0.00025$, $\lambda_3^* = 0.00030$, $\lambda_4^* = 0.00023$, $\lambda_5^* = 0.00025$ and $\lambda_6^* = 0.0002$.
5. $C_{sr} = 72$ and $C_{sp} = 48$.
6. $C_{ir} = 12$ and $C_{im} = 0.3$, for $i = 1, 2, 3, 4, 5, 6$.

By substituting the parameters of type I and type II failures in equations (10) and (11), the following equations below are obtained as follows:

$r_1(t) = 0.12t^3$. (12)

$r_2(t) = 0.06t$. (13)

$r_3(t) = 0.09t^2$. (14)

$r_4(t) = 0.003t^2$. (15)

$r_5(t) = 0.004t^3$. (16)

$r_6(t) = 0.002t$. (17)

$r_1^*(t) = 0.00132t^3$. (18)

$r_2^*(t) = 0.000875t^{2.5}$. (19)

$r_3^*(t) = 0.00012t^3$. (20)

$r_4^*(t) = 0.000805t^{2.5}$. (21)

$r_5^*(t) = 0.001t^3$. (22)

$r_6^*(t) = 0.0007t^{2.5}$. (23)

Table 1 below is obtain by substituting the assumed cost of replacement/repair and rates of type I and type II failures (equations (12) to (23)) in the replacement cost rates of the system under SARP, policy A and policy B. Table 2 is obtained as the cost of the un-planned replacement (C_{sr}) increases, and table 3 is obtained as the cost of the planned replacement (C_{sp}) decreases.

Table 1. Results obtained from evaluating the replacement cost rates of system under on SARP, policy A and policy B.

T	$\boldsymbol{C(T)}$	$\boldsymbol{CA(T)}$	$\boldsymbol{CB(T)}$
1	240.40	240.61	240.63
2	122.70	122.56	122.23
3	88.45	87.12	85.13
4	78.94	75.39	69.44
5	82.43	75.94	63.13
6	91.21	83.13	61.96
7	94.65	87.12	63.62
8	96.84	89.14	66.21
9	98.93	91.71	67.86
10	100.99	95.99	69.20

Table 2. The optimal replacement times of system under SARP, policy A and policy B as C_{sr} increases.

C_{sr}	T^*	T_A^*	T_B^*
72	4	4	6
90	4	4	5
110	4	4	5
130	3	4	5
150	3	3	5
170	3	3	4

Table 3. The optimal replacement times of system under SARP, policy A and policy as C_{sp} decreases.

C_{sp}	T^*	T_A^*	T_B^*
48	4	4	6
40	4	4	5
30	4	4	5
20	3	3	4
10	3	3	4

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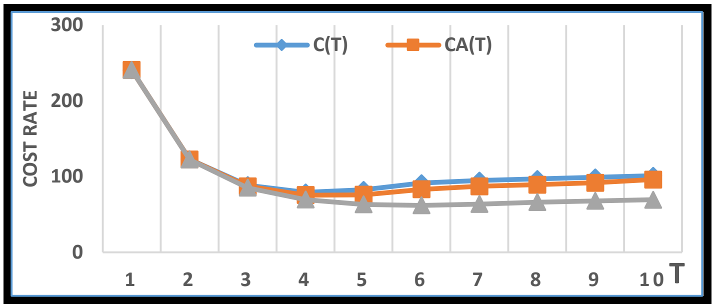

Figure 2: The plot of $C(T)$, $CA(T)$ and $CB(T)$ against planned replacement time T.

Observations from the results obtained above:

1. From table 1, observe that, the optimal replacement time of the system under SARP, is $T^* = 4$, which corresponded to $C(T^*) = 78.94$.
2. From table 1, observe that, the optimal replacement time of the system under policy A, is $T^* = 4$, which corresponded to $CA(T^*) = 75.39$.
3. From table 1, observe that, the optimal replacement time of the system under policy B, is $T^* = 6$, which corresponded to $CB(T^*) = 61.96$.
4. From table 2, observe that, as C_{sr} increases, the optimal replacement time of the system under SARP, policy A and policy B sometimes decreases.
5. From table 2, observe that, as C_{sp} decreases, the optimal replacement time of the system under SARP, policy A and policy B sometimes decreases.
6. From figure 2, observe that, $CB(T)$ is lower than $C(T)$ and $CA(T)$.

4. Discussion of the results

From the results obtained, it is observed that, preventive maintenance of the system under policy B have some advantages over SARP and policy B due to the following reasons:

1. The optimal replacement time of the series system obtained under policy B, have higher optimal replacement time than that of SARP and policy A. Thus, this will reduce the chances of early replacement of operating systems at early stage.
2. The cost of maintenance of the series system under policy B, is lower than that of SARP and policy A.

5. Conclusion

This paper considered a series system with six components, such that the system is subjected to two types of failures, which are Type I and Type II failures. We constructed three replacement cost models under SARM, policy A and policy B for the series system. A numerical example is given, and the results obtained, showed that, policy B, have some advantages over SARM and policy A. Thus, the results can be beneficial to industrial plant managers, in selecting the best maintenance policies for maintaining their plants.

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