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ON THE DYNAMICS OF MHD THERMO-PHYSICAL PROPERTIES DOUBLE-DIFFUSIVE CONVECTIVE FLUID FLOW ON A POROUS VERTICAL PLATE.

F. I. Alao, A. A. Alani and A. J. Omowaye

Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria.

Abstract

This work explored the temperature dependent thermo-physical properties of magneto-hydrodynamics double-diffusive convective fluid ow in a porous medium. The governing equations that models the transport phenomena are transformed using suitable similarity variables. The qualitative analysis (i.e. existence, uniqueness and stability of the solution) of the corresponding coupled nonlinear ordinary differential equations are established. The boundary value problem of the resulting coupled nonlinear ordinary differential equations are solved numerically using Shooting techniques together with fourth-order Runge-kutta integration scheme. The effects of various controlling parameters on velocity, temperature, and concentration distributions are showed graphically and in tabular forms. Results show that temperature dependent parameter enhanced the velocity, temperature, and concentration of the fluid. Finally, it is observed that the results of the research compare favorably with the existing ones in the literature.

Keywords: thermo-physical properties, double-diffusive, convective, MHD, fluid, porous medium.

Nomenclature

a, b constant C – concentration of the fluid C_p - Specific heat at constant pressure C_w - Concentration at the wall C_∞ - Free stream concentration D - Mass diffusivity $D(T)$ - Temperature dependent Mass diffusivity D_∞ - Ambient Mass diffusivity E_C - Modified Eckert number f – Dimensionless stream function f' - Dimensionless velocity g - Acceleration due to gravity G_m - Mass Grashoff number G_r - Thermal Grashoff number K - Thermal conductivity coefficient K_0 - Permeability parameter K_1 - Chemical reaction term K* - Forchheimer Parameter k' - Darcey Permeability k(T) - Temperature dependent thermal conductivity K_∞ - Ambient Mass diffusivity M - Magnetic parameter P – Pressure P_r - Prandtl number R_c - Modified Chemical reaction parameter S_c - Schmidt number S_w - Suction velocity T – Temperature T_w - Temperature at the wall	T_∞ - Ambient temperature u - Velocity component in x-direction U_w - Ambient velocity v - Velocity component in y-direction v_w - Ambient velocity x, y - Cartesian frame of reference Greek Symbols β_0 - Magnetic induction of the fluid β_c - Concentration expansion coefficient β_t - Thermal expansion coefficient δ - Temperature dependent parameter η - Similarity variable θ - Dimensionless temperature μ - Fluid viscosity $\mu(T)$ - Temperature dependent viscosity ρ - Density of the fluid σ - Electrical conductivity of the fluid ϕ - Dimensionless concentration ϕ_∞ - Ambient Dimensionless concentration ψ - Stream function Subscript ω - Wall Condition ∞ - Ambient Condition

Corresponding Author: Alao F.I., Email: alao@futa.edu.ng, Tel: +2348034727707, +2347039579158(AAA)

Introduction

Double-diffusive convection is a convection induced by two different densities gradients which retains distinct diffusion rates. The phenomenon of double-diffusive convection has attracted attention of many researchers during the recent past due to its occurrence in nature and industry. The root of double-diffusive convection under natural circumstances could be traced to oceanography (salt-fingers), where heat and salt concentrations occurs with different gradients and distinct diffusion rates.

These affects the variability in the input of cold freshwater from iceberg. In understanding the evolution of a number of systems that have rippled effects on density variations, double diffusive conv ection plays a major role. These include convection in the Earth's oceans, in magma chambers, and in the sun where helium and heat diffusions occurs at different rates. The analysis of double-diffusive convective fluid flow through a porous medium has been the subject of intense research activity owing to its large number of applications such as chemical contaminant dispersion all the way through oil saturated by water, geothermal reservoir, grain storage installations etc. Double diffusive convection is not limited to oceanography but, is also very important in areas such as engineering, geology, astrophysics, and metallurgy [1].

Moreover, viscous dissipation and thermo-diffusion influence on double diffusive convection past a vertical cone in a non-Darcy porous medium with variable heat and mass fluxes was studied by [2]. Narayana and Sibanda [3] worked on the Linearization method of double-diffusive convective flow in a cone. It was observed that the Dufour parameter retarded the heat transfer coefficient while increasing the mass transfer rate. Also, Soret parameter effect was to grow the heat transfer coefficient and reduction in the mass transfer coefficient. Capone [4] investigated the effects of anisotropic porous layer flow on double-diffusive penetrative convection simulated in an internal heating. Bhadauria [5] examined modulated temperature effect on the boundaries of double-diffusive convection through a porous medium. Bhadauria [6] investigated the saturated anisotropic porous layer and internal heat source effects on double-diffusive Convection. Cheng [7] analyzed the non-similar boundary layer of double-diffusive convection via a vertical truncated cone through a porous medium involving variable viscosity. It was observed that increase in the value of viscosity-variation parameter led to the decrease of the viscosity in fluid flow, thereby increasing the fluid velocity as well as the heat and mass transfer rates. Soret induced and double-diffusive convection through a shallow horizontal porous layer was studied by [8]. Kumar [9] examined thermal radiation and convective condition effects on Double-diffusive convection flow of Casson fluid. It was observed that increasing values of Biot number (Bi) radiation parameter (R), temperature ratio parameters θ_w, volumetric coefficient of thermal expansion β and buoyancy parameter λ retards the friction factor and increases the Nusselt and Sherwood numbers. Sharma and Gupta [10] examined the Darcy-Brinkman model of double-diffusive nanofluid in a rotating porous layer. It was discovered that top heavy nanofluids were so unstable that temperature at the lower layer retarded in comparison to the upper layer. Also, a lumina-water nanofluid was found to be more stable than copper-water nanofluid which in turns was more suitable than silver-water nanofluid. In the heat transfer analysis researchers had been able to discover more applications of thermal radiation in industry. These applications include gas turbines, space vehicles, nuclear power plants, hypersonic fights, etc involved in the radiation phenomenon. Recently, radiative heat transport had been showed to have a role to play in the design of renewable energy. Subhashini [11] examined the Prescribed surface temperature effect on dual solutions in a double-diffusive magneto hydrodynamics mixed convection flow adjacent to a vertical plate. Hill [12] studied the concentration based internal heat source effect on double-diffusive convection through a porous medium. It was observed that the analysis suggested a nonlinear relationship between the critical thermal and solute Rayleigh numbers. Discrete heating effect on natural convection in a vertical porous annulus was examined by [13]. Vidyanadhababu [14] worked on suction, heat and viscous dissipation effects on MHD flow of casson nanofluid. Kumar and Bhadauria [15] investigated the thermal non-equilibrium model of double-diffusive convection through a porous layer saturated involving viscoelastic fluid. It was discovered that the increasing values of porosity modified conductivity ratio γ, relaxation parameter λ_1, mechanical anisotropy parameter ξ diffusivity ratio τ, and Vadasz number (Va) was to stabilize the system. Swapna [16] carried out the effects of chemical reaction and convective condition on radiative Double-diffusive mixed convection magneto-micro polar flow past a porous medium. it was discovered that the rate of heat transfer $-\theta_1^{'}$ was strongly boosted with an increase in constant A. Also, the rate of mass transfer $-\phi_1^{'}$ was significantly enhanced with increasing values of Schmidt number C_h Sc. Many researchers had worked on MHD double-diffusive flow in a porous medium. Moreover, literature shows from the best of our knowledge that of MHD double diffusive convective fluid flow in a porous medium temperature dependent thermo-physical properties has not been considered. Thus, a reason to delve into the topic. This brings a motivation for this research work.

2. Mathematical formulation

Consider the steady 2-D flow of an incompressible MHD double-diffusive convective fluid flow in a porous medium with Temperature dependent thermo-physical properties. The x-axis is taken along vertical plate in an upward direction and y-axis is taken normal to the plate. The viscous dissipation is taking into account while the transport or heat and mass is considered with temperature dependent thermal conductivity and mass diffusion coefficient. Furthermore, the level of concentration of foreign mass is assumed to be low and as a result of the effects of soret and Dufour are negligible. Invoking the Boussinesq's approximation and the assumptions above, the governing equations can be written as follows.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial}{\partial x}\left(\mu(T)\frac{\partial u}{\partial y}\right)-\frac{\upsilon}{K'}u-\frac{b}{K^*}(u)^2-\frac{\rho}{\sigma}\beta_0^2u+g\beta_t(T-T_\infty)+g\beta_c(C-C_\infty) \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\frac{\partial}{\partial y}(K(T)\frac{\partial T}{\partial y})+\frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2 \tag{2.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D(T)\frac{\partial C}{\partial y}\right)-K_1(C-C_\infty) \tag{2.4}$$

$u=U_w(x), v=v_w(x), T=T_w, C=C_w,$

at y = 0, $u\to 0, T\to T_0, C\to C_0$, as $y\to\infty$. (2.5)

Where C_w and T_w are the concentration and temperature at the wall. C_∞ and T_∞ are the free stream concentrating and temperature respectively, u and v are the velocity components in x and y-axis, ν is the kinematic viscosity, k* is the Forchheimer, k' is the Darcy permeability, b is a constant σ is the electrical conductivity, ρ is the fluid density β_0 is the magnetic induction, g is the acceleration due to gravity β_t and β_c are thermal and concentration expansion coefficients respectively, T and C are the temperature and concentration of the fluid, C_p is the specific heat at constant pressure, μ is the dynamic viscosity, $\mu(t)$ is the temperature dependent kinematic viscosity, K (T) is the temperature dependent thermal conductivity, K_1 is the chemical reaction term, a is constant, D(T) is temperature dependent mass diffusivity. We employ the use of stream function $\psi = x\sqrt{av}\,f(\eta)$ and similarity variable $\eta = y\sqrt{\frac{a}{v}}$, to transform equations (2.1) – (2.4) subjected to boundary conditions (2.5) we have,

$$f'''-(1+\alpha^*)f'^2-(K_0+M)f'+ff''+Gr\theta+Gm\phi=0 \tag{2.6}$$

$$\theta''+\delta\theta\theta''+\delta\theta'^2+pr_\infty f\theta'+pr_\infty Ecf''^2=0 \tag{2.7}$$

$$\phi''+\delta\theta\phi''+\delta\theta'\phi'+Sc_\infty f'\phi'-Sc_\infty Rc\phi=0 \tag{2.8}$$

The corresponding boundary conditions:

$f'=1, f'=Sw, \theta=1, \phi=1$ at $\eta=0$, $f\to 0, \theta\to 0, \phi\to 0$ as $\eta=0$ (2.9)

Where $\alpha^*=\frac{bx}{K^*}$ is the Forchiemer parameter, $M=\frac{\sigma\beta_0^2}{\rho a}$ is the magnetic parameter, $K_0=\frac{\nu}{K'a}$ is the permeability parameter, $Gr=\frac{g\beta_t(T_w-T_o)}{a^2x}$ is the thermal grashof number, $Gm=\frac{g\beta_c(C_w-C_o)}{a^2x}$ is the mass grashof number, $\Pr_\infty=\frac{-\rho\nu c_p}{K_\infty}$ is the prandtl number, $Ec=\frac{a^2x^2}{K_\infty(T_w-T_o)}$ is the Eckert number, $Sc=\frac{\nu}{D_\infty}$ is the Schmidt number, $Rc=\frac{K_1}{a}$ is the modified chemical reaction parameter.

3. Numerical Procedure

The set of nonlinear coupled ordinary differential equations (2.6) – (2.8) constrained by the boundary conditions (2.9) are evaluated arithmetically with classical Runge-Kutta scheme via shooting technique. The computations were carried out using MATLAB software. The convergence criterion is set to 10^{-4} with the step size of $\Delta\eta=0.001$. we substitute 10 for the asymptotic boundary conditions in equation (2.9) to obtain the similarity variable $\eta_{\max}$, thus

$\eta_{\max} = 10,\ f'(10) = 0(10) = \phi(10) = 0$

The preference of $\eta_{\max}$ = 10 guaranteed that the entire numerical result attain the asymptotic estimates accurately. The present result is compared with previously existing data in the literature by [17]. There exists a clear distinction between the two cases under consideration with respect to the ones earlier reported as Pr increases.

4. Result and discussion

Detailed calculations have been executed for better understanding of the physical case. The numerical results are presented in graphs as velocity, temperature and concentration profiles. For our numerical calculations, the transformed nonlinear ordinary differential equations coupled with its boundary conditions depend on the following controlling parameters; M, Pr, Sc, Rc, Sw, Gr, Ec, Gm, K0 Here, we aim to examine the effects of the above-mentioned controlling parameters on non-dimensionalized velocity, temperature and concentration.

Figure 4.1 exhibit the effect of the applied magnetic field on the velocity, temperature and concentration distributions. The imposed magnetic field on the flow gives rise to a resistive force called Lorentz force. Lorentz force slows down the motion of an electrically conducting fluid. This is evidence in the moment equation (2.2). This fact is obvious in the velocity and concentration distributions as shown in figure 4.1(a) and 4.1(c). It is interesting to see that increasing the magnetic parameter increases the temperature distribution.

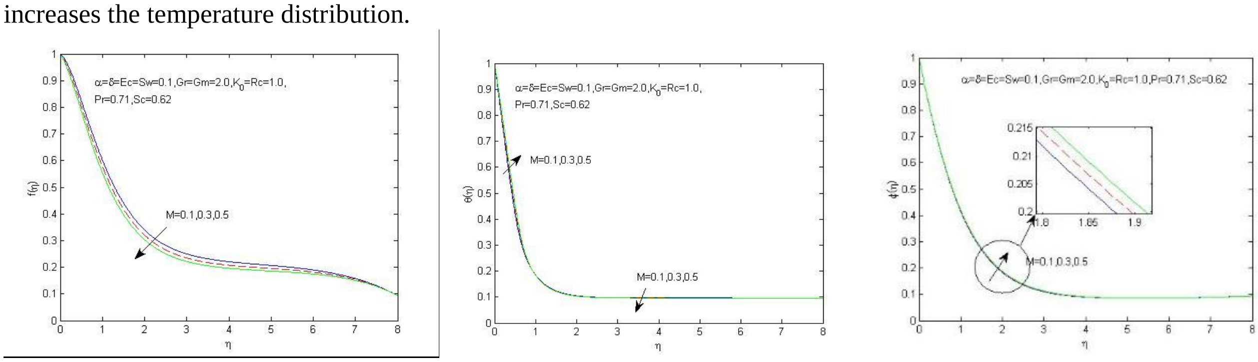


Figure 4.1(a) Figure 4.1(b)F Figure 4.1(c)

Figure 4.2 exhibit the effect of prandtl number[Pr] on the velocity, temperature and concentration fields. It is observed from Figure 4.2[a], 4.2[b] and 4.2[c] that increasing the prandtl number decreases both the velocity, temperature sand concentration fields very close to the plate. The reason behind this is that at higher prandtl number implies that the fluid has relatively low thermal conductivity.

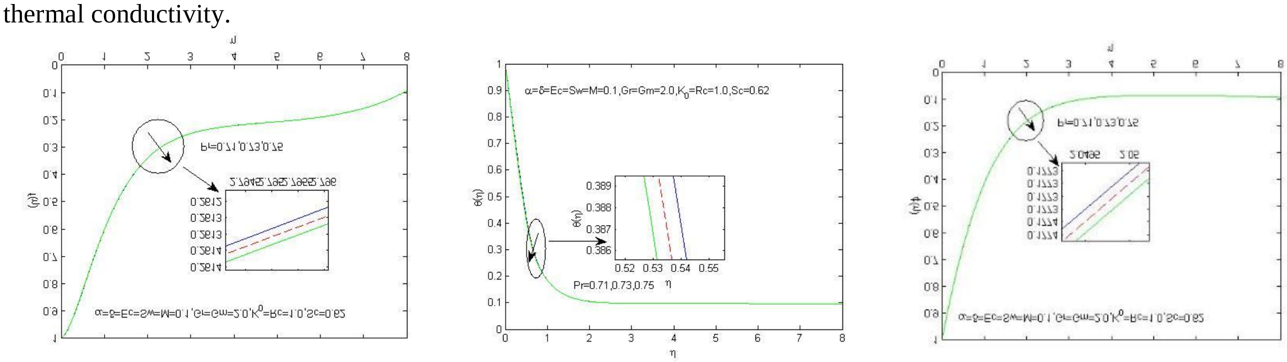


Figure 4.2(a) Figure 4.2(b) Figure 4.2(c)

Figure 4.3 shows the effect of the chemical reaction parameter [Rc] on the velocity, temperature, and concentration plots. It is interesting to see from Figure 4.3(a) and 4.3(c) that increase in the chemical reaction parameter [Rc] increases the velocity and concentration plots while the thermal plot retards. The chemical reaction could be reversible or irreversible. It has a great effect on the solutal concentration when the fluid reaction is increased chemically, it accelerates the fluid velocity and concentration

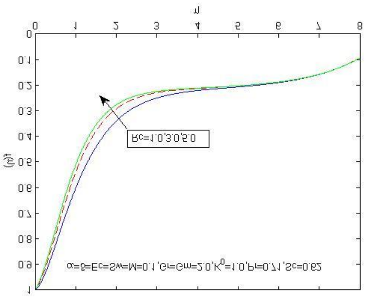


Figure 4.3(a)

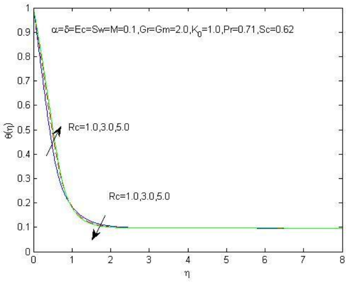


Figure 4.3(b)

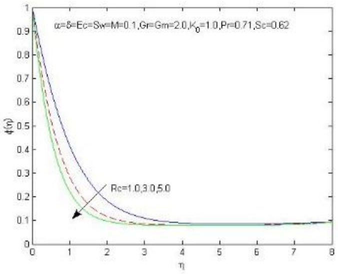


Figure 4.3(c)

Figure 4.4 illustrates the effect of Schmidt number [Sc] on velocity, temperature, and concentration profiles. The result in Figure 4.4(a) and 4.4(c) reveals that increasing Schmidt number [Sc] intensifies the temperature profile slightly, while it retards the velocity, as well as the concentration profile.

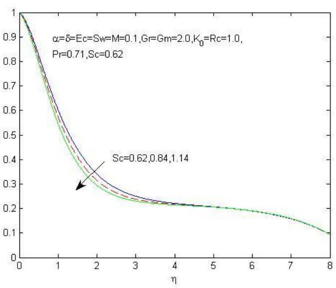


Figure 4.4(a)

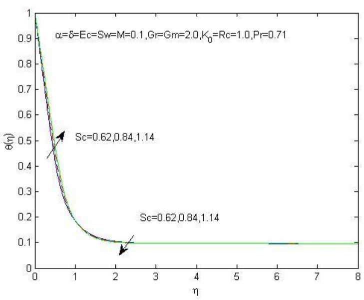


Figure 4.4(b)

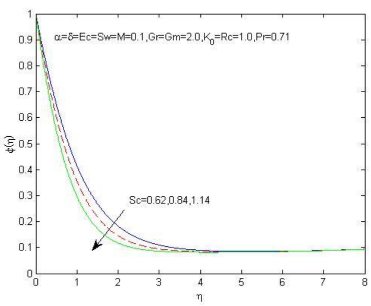


Figure 4.4(c)

In Figure 4.5 the effect of thermal Grashof number Gr on velocity, temperature, and concentration graphs is displayed. The thermal Grashof number Gr is a dimensionless number that defines the ratio of the buoyancy to the viscous force acting on the fluid. It is observed from Figure 4.5(a) that as the thermal Grashof number increases, the velocity graph is increased while the temperature and concentration retards. At higher thermal Grashof number, the boundary layer flow is laminar and vice versa. The buoyancy force has great effect on the flow and drastically increase the fluid velocity.

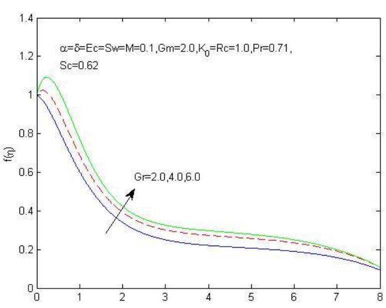


Figure 4.5(a)

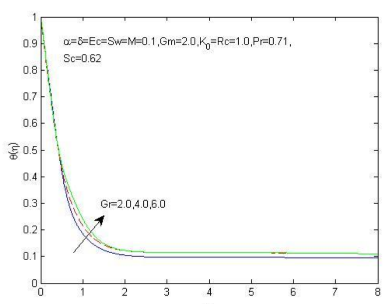


Figure 4.5(b)

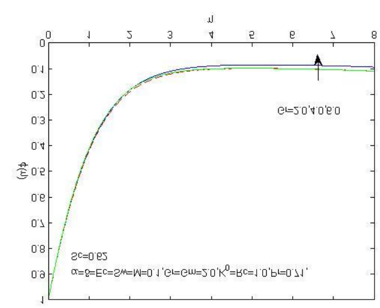


Figure 4.5(c)

In figure 4.6, the effect of mass Grashof number Gm on velocity, temperature, and concentration graphs is displayed. The mass Grashof number Gm is a dimensionless number that defines the ratio of the buoyancy to the viscous force acting on the fluid. It is observed from Figure 4.6(a) that as the mass Grashof number increases, the velocity graph is increased while the temperature and concentration retards. At higher mass Grashof number, the boundary layer flow is laminar and vice versa. The buoyancy force has great effect on the flow and drastically increase the fluid velocity.

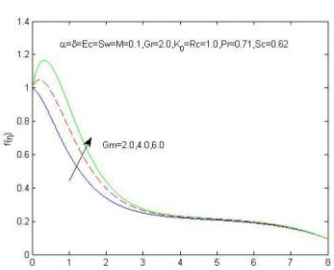


Figure 4.6(a)

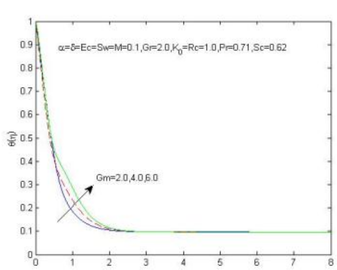


Figure 4.6(b)

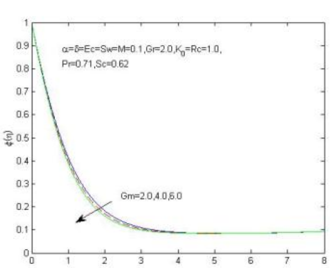


Figure 4.6(c)

Figure 4.7 displayed the effect of viscous dissipation term known as Eckert number Ec on the velocity temperature and concentration plots. The Eckert number depict Intensifying Eckert number Ec gives rise to temperature as well as velocity in the boundary layer.

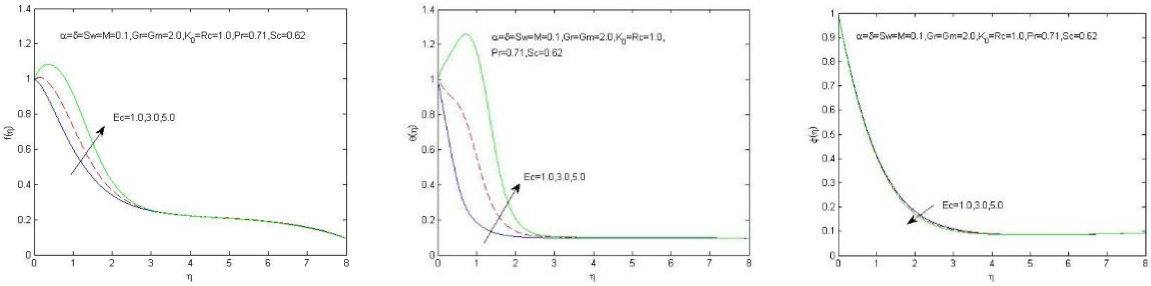


Figure 4.7(a) Figure 4.7(b) Figure 4.7(c)

Figure 8 exhibits the effect of permeability parameter K_0 on the velocity, temperature and concentration profiles. It is observed from Figures 4.8(b) and 4.8(c) that increasing K_0 increases the temperature and concentration profiles. From Figure 4.8(a) it is interesting to see that increasing the permeability parameter retards the velocity profile.

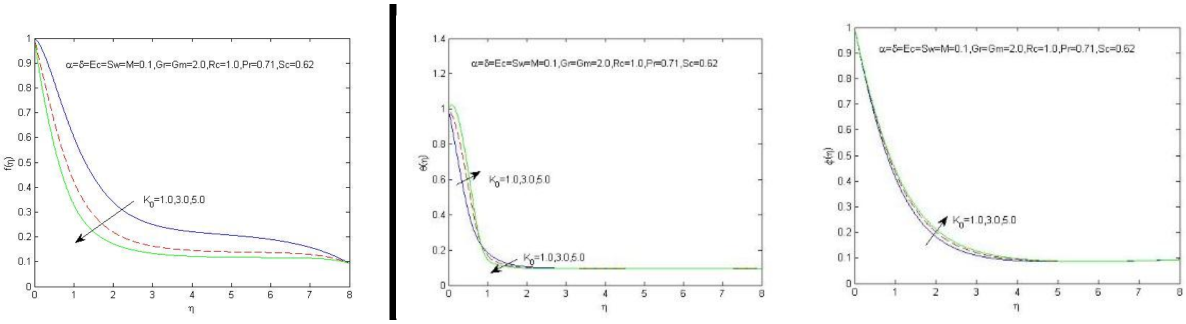
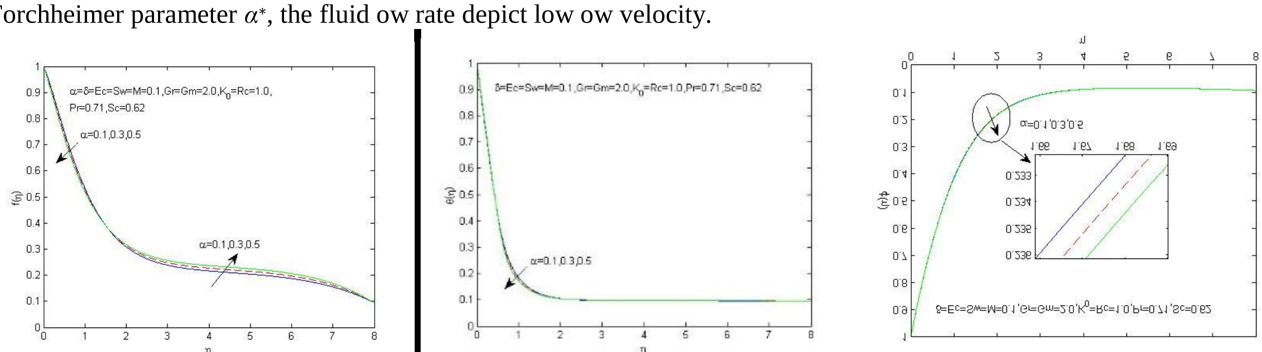


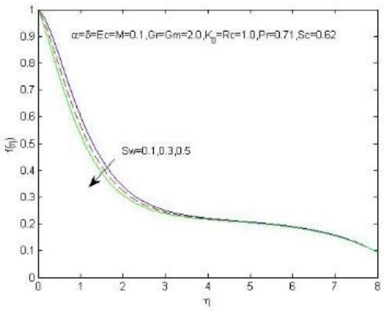
Figure 4.8(a) Figure 4.8(b) Figure 4.8(c)

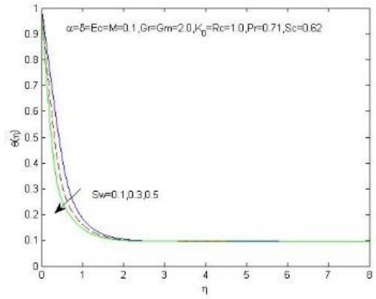
In Figure 4.9, the effect of Forchheimer parameter α^* on velocity, temperature, and concentration graphs is displayed. The Forchheimer parameter α^* is the fluid flow rate that is directly proportional to the pressure gradient, is shown to be accurate only at low flow velocities, both in liquid and gas phase systems. It is observed from Figure 4.9(a) and 4.9(c) that as the Forchheimer parameter increases, the velocity and concentration graphs retards while the temperature intensifies. At higher Forchheimer parameter α^*, the fluid ow rate depict low ow velocity.

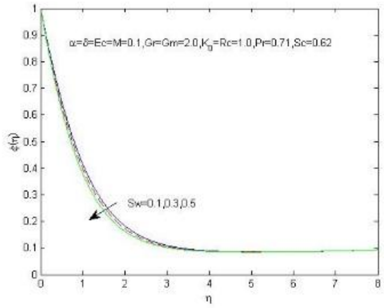


[redacted] Figure 4.9(a) Figure 4.9(b) Figure 4.9(c)

Figure 4.10 shows the effect of the suction velocity S_w on the velocity, temperature and concentration fields. It is noticed that increasing the suction velocity increases temperature field but retards velocity and concentration fields.

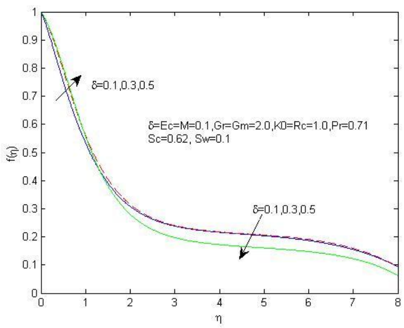


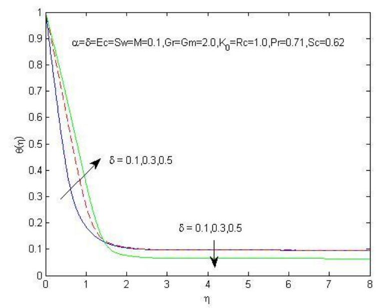




[redacted] Figure 4.10(a) Figure 4.10(b) Figure 4.10(c)

Figure 4.11 depicts the effect of the temperature dependent parameter δ on the velocity, temperature and concentration fields. It is found that increasing the temperature dependent parameter δ intensifies the velocity and concentration fields but retards the temperature.





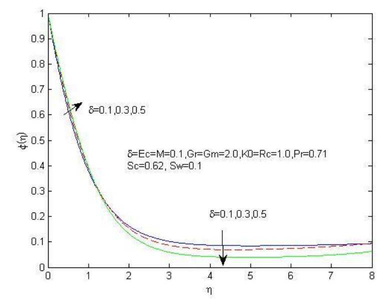


Figure 4.11(a) Figure 4.11(b) Figure 4.11(c)

5 Conclusions

The effects of temperature dependent thermos-physical properties on MHD double diffusive fluid ow in a porous medium have been examined. The dimensional partial differential equations controlling the MHD fluid ow are transformed to dimensionless equations with the help of suitable similarity variables. The resulting nonlinear coupled ordinary differential equations are evaluated numerically with classical Runge-Kutta method via shooting technique together with its boundary conditions. The main features of the problem are analyzed and discussed qualitatively. Suitable results are presented graphically with respect to variation in the controlling parameters on f, θ, and ϕ. In the variable temperature dependent thermos-physical properties model, the following important conclusions are drawn:

1. increase in the magnetic M parameter retards both the velocity and temperature plots but intensifies the concentration plot;
2. the suction velocity S_w retards both the velocity and concentration profiles but intensifies the temperature pro le;
3. increase in the chemical reaction parameter R_c intensifies both the velocity and concentration distributions but retards the temperature distribution; and
4. increase in the temperature dependent parameter δ increases both velocity and concentration plots but retards the temperature plot.

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