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COMPARATIVE ANALYSIS OF POPULATION MODEL USING DISCRETE AND NATURAL MODEL: CASE STUDY OF *DELTA STATE POLYTECHNIC, OGWASHI-UKU*

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Abstract

This paper focuses on literature reviewed of Population modelling, Principles of Mathematical Modeling and Comparative Analysis of Population growth model. The population growth of students of Delta State Polytechnic, Ogwashi-Uku, comparing the growth rate by Geometric and Natural Growth model. The data of admission of students are collected from the Admission centre in Delta State Polytechnic, Ogwashi-Uku which acts as the basis of input to generate the output from the software developed to forecast using the Geometric and Natural Growth Model. The comparative analysis showed 0.2% difference between Geometric and Natural Growth Model. This shows that both models are capable to predict the student's admission growth rate. The Natural Growth and Geometric Increase Method is a simple realistic population model based on past information. The Geometric Increase method tends to give a higher estimate than normal since it behaves exponentially. It more accurately describes the continuous and cumulative nature of population growth. In normal practice, an average of the arithmetic method and geometric method is performed to get a more accurate estimate.

Introduction

Forecasting of population can be accomplished with different mathematical methods by using present and past population records that can be obtained from local census offices and past organizational population. These mathematical methods are generally classified in two categories:

Short term methods (1-10 years). The following are methods for the estimation of short term of population.

Arithmetic progression
Geometric progression
İller Bankası method
Decreasing rate of increase
Graphical extension

Long term methods (10-50 years). The following are the methods for estimation of long term of population.

Comparative method
Ratio & correlation method
Component method
Logistic method (mathematical curve fitting)

A population model is a type of mathematical model that is applied to the study of population dynamics, [1].Models of population dynamics (hereafter, referred to as population models) are useful tools for understanding, explaining, and predicting the dynamics and persistence of biological populations. From a management perspective, such models can be used for assessing the status of a population, diagnosing causes of population declines or explosive growth, prescribing management targets, and evaluating the prognosis of a population's likely responses to alternative management actions [2]. Population modelling played an important role in reversing the decline of the endangered loggerhead sea turtle population in the United States [3]; [4]; [2]. Until the 1980s, sea turtle conservation efforts had focused on the protection of nests, eggs, and hatchlings on nesting beaches.

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Models of population dynamics can also help to predict populations' responses to environmental changes, such as global climate change. Global climate change is predicted to influence arctic sea ice adversely, and this could affect the population dynamics and persistence of species that depend on sea ice environments. For example, polar bears depend on arctic sea ice for feeding and breeding. By integrating field data, climate-change models, and population models,[5] predicted that the polar bear population in the southern Beaufort Sea would experience a drastic decline because of a reduction in sea ice extent by the end of the 21st century

Population models are also useful tools in the management of overabundant species. For example, the American bullfrog is an introduced species on Vancouver Island and is adversely affecting biodiversity on parts of the island. A modelling study by [6] reported that the management strategy of targeting removal of tadpoles may not be effective because partial removal of tadpoles could lead to higher tadpole survival owing to reduced density-dependent effects. Their results revealed that culling metamorphosis in fall would be most effective in controlling bullfrog populations. A theoretical study by [7] suggested that control of overabundant species by harvest (or removal) could backfire because populations of species characterized by early maturity and high fecundity may experience rapid growth after harvest or removal as a result of density-dependent overcompensation. Other examples of the application of population models include assessing the influences of culling and fertility control on the population dynamics of an overabundant of population Bradford and Hobbs, [8] and controlling the fertility of the koala on koala forest dynamics,[9], evaluating the efficacy of euthanasia versus trap-neuter-return for management of free-roaming cats Andersen et al.[10] and the efficacy of fertility control in a white-tailed deer population Merrill et al., [11], discerning mechanisms underlying a recent rapid population growth in yellow-bellied marmots,[12], predicting effects of El Niño on the dynamics and

Accurate population census of any country is very important because it enables the government and her institutions to plan effectively for her citizens with regards to their well-being" [13]. Comparability is a core demographic value, and to understand the limits of the comparability of census data across time and space. In reality, all planning by the government begins with population structure and forecasts. It is imperative that any responsible government should have current and reliable population forecast for better economic planning. Over the years, there have been concerns about the appreciation of governments on the need for reliable and accurate population census on budgeting and planning for the welfare of her citizens. In [14] explained the controversy surrounding the recently concluded population census in Nigeria arguing that the 2006 population census was no way different from the past falsified ones held in Nigeria. [14] in his study used regression techniques to test the association from 1980 to 2003 to ascertain the validity of the assumed inverse association between population's growth and the development in Nigeria.

The issues variously thrown up were on accuracy and reliability of our population forecasting exercises over the years which have led to the outright cancellations of some past census exercises census exercise in (1962 and 1973) comes up. These recurring issues stated above motivated this comparative study of population models to avert challenges encountered by researchers and society at large .It is imperative to draw a comparison between the two different population models respectively. This will help to appreciate any possible inadequacies of the past and to address such in the proposed exercise in the future.

This research hopes to study the development of a computer aided design for comparative analysis of natural growth model and discrete population model with the specific mindset of providing an interactive arithmetic and discrete population model to estimate the population of a given area, to evaluate natural and discrete model and determine the most effective.

It is imperative to consider literal works related to the research topic. The use of mathematical models to analyse population census will be emphasized. In-depth study of the natural and discrete model is carried out and the models are adopted for the implementation of the computer aided software,

Literature Review

Mathematical Modeling is the process by which we formulate and analyze model equations and compare observations sto the predictions that the model makes, [15].

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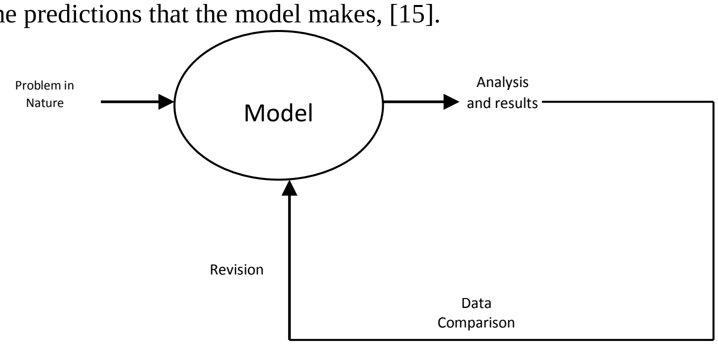

Figure 1: The Process of Mathematical Modeling[15].

Principles of Mathematical Modeling

By a mathematical model we understand an equation, or a set of equations, that describe some phenomenon that we observe in science, engineering, economics, or some other area, that provides a quantitative explanation and, ideally, prediction of observations, [15]. [15] stated that a good model should have predictive powers, a model based on available observations gives correct answers in other cases: General Theory of Relativity, light deflection, perihelio precession of Mercury, gravitational waves, dirac equations (existence of positrons).Contains earlier working models as sub-cases.

Population Growth and Economic Development

The connection between population growth and economic development has been a much debated topic ever since the world population passed the two billion mark, [16] stated that a vigorous debate on the relationship between those two factors has been going on in all countries, irrespective of whether they have less developed economies, developed economies or transition economies. Many theories abound, and the first theory states that population growth stimulates economic growth. The second theory views population growth as a phenomenon that adversely affects economic growth. That means the relationship between population growth and economic development can be measured by looking at the impact of population growth on economic development and vice-versa. It is clear that in the past economists and demographers considered the inter-relationship between population growth and economic development from both an optimistic perspective as well as from a pessimistic perspective, [17]**.**

Population Growth and Economic Development

First, we will examine the effect of population growth on the economic development of a country. On the one hand, through rapid population growth, there will be some economic development. Further, the economy will also be controlled by a large population because a large market has to be supplied. This market will attract the future industrialists who will utilize the most technologically advanced methods to obtain the maximum yield out of the available resources. In this manner the growing population will speed up the economic growth in a country. For example, Japan can be cited as a country with a high population but it managed to achieve a high living standard by developing the economy. But when we consider some countries like India, it is clear that the growing population is a big problem that affects the economic development of that country. In this case, the steadily growing population seems to be a hindrance to the country's economic development. However, when a country builds up a more robust economy, the birth rate usually begins to decline. That happens because economic progress makes people realize the appropriateness of having smaller families and also drives them to take an interest in planning their families. The reason for this is economic growth is always accompanied by a rise in prices and increase in the cost of living. Then children are considered not as a treasure by parents but as a burden for them. The prevalence of small families and low death rates are special characteristics of this period.

The study by [18] has also concluded that population size and density have a transitional impact on economic growth. Other studies have reported that the shift in age distribution pattern has had a significant impact on economic growth through savings and investments [19]; , [20]. Thus improve their economy as posited. [21] mentions that Malthus and neo-Malthusians believe that population growth is negatively correlated with economic growth. But Julian Simon argues that the correlation is positive. According to [22], the newer generation of job-seekers who enter the workforce will have better education than the previous generation of workers, and so the former will be more productive than the latter, leading to greater economic development.

Conclusively, It is clear that in the past, economists and demographers considered the inter-relationship between population growth and economic development in an optimistic manner as well as in a pessimistic way. When considering those with an optimistic outlook, they adopted a welcome attitude towards population increase –that is, they considered it was not necessary to limit the population of a country. But the pessimists express the view that if a country is to reach a proper state of development, the speed of population growth should be reduced. According to the foregoing discussion, it is clear that although a steadily growing population might appear to be a hindrance to a country's economic development, most of the countries have in practice accepted their population as a blessing.

Comparative Analysis of Population

This study brings out a clearer picture of the population dynamics in Africa especially for the population growth rate which has been a crucial factor and also a major challenge in the proper planning and budgeting for the well-being of the citizenry. Four growth models are employed namely; arithmetic geometric, exponential and logistic models. Results obtained showed some interesting features. Accurate population census of any country is very important because it enables the government and her institutions to plan effectively for her citizens with regards to their well-being"[16]. Comparability is a core demographic value, and to understand the limits of the comparability of census data across time and space, it is important to recognize if, how and why, concepts and definitions change between censuses. Virtually, all planning by the government begins with population structure and forecasts. It is imperative that any responsible government should have current and reliable population forecast for better economic planning. Over the years, there have been concerns about the appreciation of governments on the need for reliable and accurate population census on budgeting and planning for the welfare of her citizens.

Comparative Analysis of Population Censuses in Africa

As [23] asserted that these issues variously thrown up were on accuracy and reliability of our population census exercises over the years. These issues led to the outright cancellations of some past exercises (1962 and 1973) and in some cases due dates were postponed (1983 and [13]). In order to draw our attention to some of these issues, particularly as another census year, 2017 approaches, it is imperative to draw a comparison between the World Bank figures with that of countries in African regions respectively. This will help to appreciate any possible inadequacies of the past and to address such in the proposed exercise in the future. "These recurring issues motivated this comparative study of population censuses among the five regions of Africa using four different growth models (Arithmetic, Geometric, Exponential, and Logistic) in comparison while the World Bank data serve as standard"[16].The use of mathematical models to analyze population census was emphasized by [24], where he stated that Mathematical model is very important for the estimation of population projections. He stated further that, mathematical model is essentially an endeavour to find out structural relationships and their dynamic behaviour among various elements in demography. In a related development, [25] and [26] demonstrated that simple mathematical models account for growth-increase or decline-in human population. Many authors have proposed several models for forecasting population censuses. In Logistic growth model, Pearl and Reed [25], showed how changes in mortality, fertility and agricultural productivity actually have distinct effects on the population growth rate and equilibrium. [26], applied Logistic models in combination with the Fisher-Pry transform technique to provide clear and suggestive outputs for supporting medium and long-term forecasting of technology changes. In [27], Exponential and Logistic growth model was used to model the population growth of Ghana using data from 1960 to 2011. The Exponential model predicted a growth rate of 3.15% per annum and also predicted the population to be114.82million in 2050 while the Logistic model predicted a growth rate of 5.23% per annum and the population of Ghana to be 341.24million in 2050.

Methods

The two method of population growth models, natural and geometric model was employed. The data were obtained using the different models and subsequently used to compute and forecast the population values for the twenty years. The two growth models are:

Natural Growth Model

Malthus' model is commonly called the **natural growth model** or **exponential growth model**. For this model we assume that the population grows at a rate that is proportional to itself. If P represents such population then the assumption of natural growth can be written symbolically as

$$P_{n+1} = P_n e^{rt} \quad (1)$$

where:
P = is the population
r = is the rate of population growth
t = time (number of years)
e = exponential constant (2.718).

Geometric Model

Geometric extrapolation is desirable for short intervals, simple methods and when forecasting for a new city or an institution.

Geometric rates are preferable to arithmetic rates for the extrapolation of decreases in population over a series of years

The basic model for geometric change in population size is:

$$P = P_o \lambda^t \quad (2)$$

This is based on the hypothesis that rate of change of population is proportional to the population. According to this, method it is assumed that the rate of increase of population growth in a community is proportional to the present population.

P_o denotes initial size,
P denotes population at time t
t denotes time (measured in decades)
λ is the 'finite population multiplier' which can be interpreted as $\lambda = e^i$ for continuous change or $\lambda = 1 + i$ for discrete (constant) 'compound interest' or 'birth-pulse' populations.

$\mathbf{P = P_o\lambda^t}$
$\mathbf{\lambda = (1+ i)}$ for discrete change
Therefore $\mathbf{P = P_o (1+i)^t}$ where,
P_0 : Initial population size
P: Population size at time t
i: Average percentage increase per decade
t: Number of decades

- $\frac{dP}{P} = i\,.dt$
- $\int \frac{dP}{P} = \int i\ .dt$
- $\ln P = it + c$
- (When t = 0, $P = P_o$, therefore $c = \ln P_o$)
- $\ln P = it + \ln P_o$
- $\ln P - \ln P_o = it$
- $(\ln P - \ln P_o = \frac{\ln P}{\ln P_o})$
- $\frac{\ln P}{\ln P_o} = it$
- $\frac{P}{P_o} = e^{it}$
- $P = P_o e^{it}$

Data Collection: The data were collected from the Admissions Department in Delta State Polytechnic, Ogwashi-Uku. The number of students admitted in 2017/2018 through 2019/2020 academic session were used are previous and present population data which served as input for forecasting using the developed visual basic software.

ANALYSIS, OBSERVATIONS AND FINDINGS

Table 1.0: Population Forecast Using Geometric Growth Model

Initial Population	Present Population	Average% Increase	Number of Years	Population Forecast
3396	3596	5.561735261	1	3801.644372
3396	3596	5.561735261	2	4019.048924
3396	3596	5.561735261	3	4248.886185
3396	3596	5.561735261	4	4491.867145
3396	3596	5.561735261	5	4748.743451
3396	3596	5.561735261	6	5020.309737
3396	3596	5.561735261	7	5307.406078
3396	3596	5.561735261	8	5610.920591
3396	3596	5.561735261	9	5931.792182
3396	3596	5.561735261	10	6271.01345
3396	3596	5.561735261	11	6629.633756
3396	3596	5.561735261	12	7008.762474
3396	3596	5.561735261	13	7409.572418
3396	3596	5.561735261	14	7833.30347
3396	3596	5.561735261	15	8281.266421
3396	3596	5.561735261	16	8754.84702
3396	3596	5.561735261	17	9255.510261
3396	3596	5.561735261	18	9784.80492
3396	3596	5.561735261	19	10344.36834
3396	3596	5.561735261	20	10935.9315

Table 2.0: Population Forecast Using Natural Growth Model

Initial Population	Rate of Increase	Number of Years	Exponential	Population Forecast
3596	0.005	1	2.718	3614.023151
3614.023151	0.005	2	2.718	3650.340903
3650.340903	0.005	3	2.718	3705.502978
3705.502978	0.005	4	2.718	3780.351264
3780.351264	0.005	5	2.718	3876.041265
3876.041265	0.005	6	2.718	3994.071872
3994.071872	0.005	7	2.718	4136.324538
4136.324538	0.005	8	2.718	4305.11329
4305.11329	0.005	9	2.718	4503.24743
4503.24743	0.005	10	2.718	4734.10932
4734.10932	0.005	11	2.718	5001.750248
5001.750248	0.005	12	2.718	5311.00817
5311.00817	0.005	13	2.718	5667.6521
5667.6521	0.005	14	2.718	6078.559128
6078.559128	0.005	15	2.718	6551.931594
6551.931594	0.005	16	2.718	7097.563892
7097.563892	0.005	17	2.718	7727.17084
7727.17084	0.005	18	2.718	8454.792722
8454.792722	0.005	19	2.718	9297.296106
9297.296106	0.005	20	2.718	10274.99474

From the tables above, changes in population forecast has been observed. The higher the year, the more the population maintain the same population rate of increase and average percentage increase respectively.

Table 3.0: Comparison of Values of Natural Growth Model and Geometric Growth Model.

S/N	Number of Years	Geometric Population Forecast (GPF)	Natural Growth Population Forecast (NGPF)	Δ = (GPF - NGPF)
1	1	3801.644372	3614.023151	187.6212205
2	2	4019.048924	3650.340903	368.7080208
3	3	4248.886185	3705.502978	543.3832072
4	4	4491.867145	3780.351264	711.5158805
5	5	4748.743451	3876.041265	872.7021861
6	6	5020.309737	3994.071872	1026.237865
7	7	5307.406078	4136.324538	1171.08154
8	8	5610.920591	4305.11329	1305.807301
9	9	5931.792182	4503.24743	1428.544752
10	10	6271.01345	4734.10932	1536.904129
11	11	6629.633756	5001.750248	1627.883508
12	12	7008.762474	5311.00817	1697.754304
13	13	7409.572418	5667.6521	1741.920318
14	14	7833.30347	6078.559128	1754.744342
15	15	8281.266421	6551.931594	1729.334827
16	16	8754.84702	7097.563892	1657.283128
17	17	9255.510261	7727.17084	1528.339422
18	18	9784.80492	8454.792722	1330.012199
19	19	10344.36834	9297.296106	1047.072236
20	20	10935.9315	10274.99474	660.9367668

Percentage Error (% Error)

Here, Percentage error is calculated as:

$$(\%error) = \frac{ExperimentalValue - TheoriticalValue}{Theoriticalvalue} * 100 \quad (4)$$

Our experimental value in this research is the Geometric Population Forecast value while the theoretical value is our Natural Growth Population values.

Therefore, percentage error is calculated thus:

$$(\%error) = \frac{NGPF}{GPF - NGPF} * 10 \quad (5)$$

$$\textbf{Recal}: (Error) = \frac{GPF - NGPF}{} \quad (6)$$

$$\%Error = \frac{\textbf{Error}}{NGPF} * 100 \quad (7)$$

Example:

$$\frac{187.6212205}{3614.023151} * 100 = 0.0512 * 100 = \mathbf{5.192\%}$$

Table 4.0: Percentage Error

S/N	No. of Years	(GPF)	NGPF	Error = (GPF - NGPF)	Error / NGPF	%Error
1	1	3801.64437	3614.023151	187.6212205	0.051914781	5.191478%
2	2	4019.04892	3650.340903	368.7080208	0.101006462	10.10065%
3	3	4248.88619	3705.502978	543.3832072	0.146642227	14.66422%
4	4	4491.86714	3780.351264	711.5158805	0.188214224	18.82142%
5	5	4748.74345	3876.041265	872.7021861	0.225152966	22.5153%
6	6	5020.30974	3994.071872	1026.237865	0.25694026	25.69403%
7	7	5307.40608	4136.324538	1171.08154	0.28312129	28.31213%
8	8	5610.92059	4305.11329	1305.807301	0.303315433	30.33154%
9	9	5931.79218	4503.24743	1428.544752	0.317225463	31.72255%
10	10	6271.01345	4734.10932	1536.904129	0.324644833	32.46448%
11	11	6629.63376	5001.750248	1627.883508	0.325462773	32.54628%
12	12	7008.76247	5311.00817	1697.754304	0.319667048	31.9667%
13	13	7409.57242	5667.6521	1741.920318	0.307344256	30.73443%
14	14	7833.30347	6078.559128	1754.744342	0.288677679	28.86777%
15	15	8281.26642	6551.931594	1729.334827	0.263942748	26.39427%
16	16	8754.84702	7097.563892	1657.283128	0.233500276	23.35003%
17	17	9255.51026	7727.17084	1528.339422	0.19778771	19.77877%
18	18	9784.80492	8454.792722	1330.012199	0.15730867	15.73087%
19	19	10344.3683	9297.296106	1047.072236	0.112621156	11.26212%
20	20	10935.9315	10274.99474	660.9367668	0.064324779	6.432478%

Average Percentage Error

$$Average\ \%error = \frac{\textbf{Sum of Percentage Error}}{\textbf{n}} = \mathbf{446.\frac{88}{20} = 22.\frac{344}{100} = 0.22\%}$$

Therefore, the average percentage error from values between the Geometric Growth and Natural Growth Model is: **= 0.2%**

A percentage very close to zero means shows accuracy. It is always necessary to understand the causes of the error.

The plot in figure 1.0 below depicts a plot of Geometric Progression model values against number of years. Here it is observed that there is a continuous increase of population. Here, the rate of increase in population is directly proportional to years.

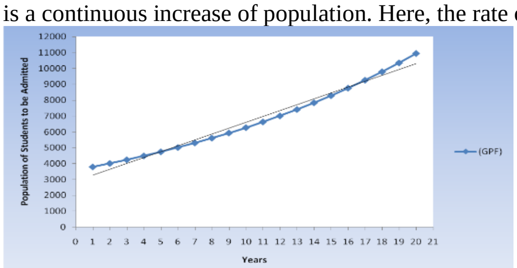

Figure 1.0: Graph of Geometric Growth Model of Population of Students to be admitted against years.

Figure 2.0 below also depicts a plot of Natural Growth Model values against years. Here also, it is observed that there is a continuous increase of population with respect to years. The rate of increase in population is also directly proportional to number of years.

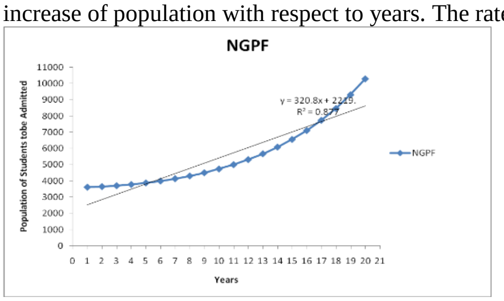

Figure 2.0: Graph of Natural Growth Model of Population of Students to be admitted against years.

Here, it has been observed that the Natural Growth and Geometric Increase Method is a simple realistic population model based on past information. The Geometric Increase method tends to give a higher estimate than normal since it behaves exponentially. It more accurately describes the continuous and cumulative nature of population growth. In normal practice, an average of the arithmetic method and geometric method is performed to get a more accurate estimate.

Conclusion

The introduction of forecast in the future population has actually helped in planning against all future odd, including economic breakdown. The need for the computerization of the organization was highly emphasized as computer could store, update, and retrieve information in a manner that no human agent can do. Computer could always process data and produce accurate and reliable results when given correct data as an input. The use of computer in forecast operations will solve a lot of problems. If regulative mechanisms have not time lag in their reaction onto population size changing, and productivity of individuals is constant there are no oscillation regimes for population, and population stabilizes asymptotically at unique level for all positive initial values. If regulative mechanisms have a time lag chaotic and oscillation regimes can be observed for population even with constant productivity. Constructing of models with discrete

time is based on the assumption that coefficient of population birth rate (it is determined as relation of population sizes or densities of two nearest generations) can be presented as rather simple function with respect to population size.
Finally, the paper has contributed to body of knowledge by coming up with multiple regression for Geometric population forecasting (GPF) and Natural growth population forecasting (NGPF) as depicted in the Figure 1.0 and Figure 2.0 for prediction of student's population.

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