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THE POWER SERIES APPROXIMATION OF THE THIRD DERIVATIVE BLOCK METHOD FOR THE SOLUTION OF HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS

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Abstract

In this research, we propose a approximation of third derivative for the direct solution of higher order ODEs. The method was derived via interpolation and collocation by the introduction of off-mesh point at both grid and off-grid, using the power series. The analysis of the method was examined, and it was found to be consistent, zero-stable, convergent and absolutely stable. The method was tested on three highly stiff problems and from the results obtained, the method are more accurate than the existing methods. We further sketched the solution graph of this method and it is evident that the new method convergence toward the exact solution.

Keyword: Third derivative, ODEs, Interpolation and collocation, Off-mesh point, Power series, Highly stiff problem.

1 INTRODUCTION

In this research, we considered solving initial value problems (IVPs) for third order ordinary differential equations (ODEs) in the form:

$$y''' = f(x, y, y' y''),\ y(a) = \alpha,\ y'(a) = \beta,\ y''(a) = \gamma,\ x \in [a, b] \tag{1.1}$$

Equation (1.1) has been practically used in a wide variety of applications especially in science and engineering field and some other area of applications. The reduction of (1.1) to the system of first-order equations will leads to computation cost. Several researchers such as [1]-[7] have investigated and suggested the best approach for solving the system of higher order ODEs directly.

An efficient one step block method with generalized two-point-hybrid is developed for solving initial value problems of third order ordinary differential equations directly has been proposed by [8],[9] developed one-step block method with four equidistant generalized hybrid points for solving third order ordinary differential equation using a set new generalized linear block method. In [10] has proposed two-point four step direct implicit block method is presented by applying the simple form of Adams- Moulton method for solving directly the general third order ordinary differential equations (ODEs) using variable step size [11] proposed the computation of an accurate block method with step-length of eight for solving third order ordinary differential equations also is [12]

In order to overcome the challenges in reduction method, scholars such as [13], [16] developed block methods for direct solution of higher order ODEs where the accuracy of the methods is better than when it is reduced to system of first order ODEs.

The essential aim of this research is to propose a new approximation of third derivative block method for the direct solution of third order ODEs of the form (1.1) using power series method as

a basic function. The properties of the proposed block method is been examined, while the errors from the propose method is compared with that of existing methods

2. PRELIMINARY

2.0 Derivation of the Scheme

We consider a power series approximate solution in the form:

$$y(x) = \sum_{j=0}^{p+q-1} a_j x^j \tag{2.1}$$

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where p and q are number of distinct collocation and interpolation respectively. Substituting the third derivative of (2.1) into (1.1) gives:

$$\sum_{j=0}^{p+q-1} j(j-1)(j-2)a_j x^{j-3} = f(x, y, y', y'') \tag{2.2}$$

interpolating (2.1) at point $x = x_{n+\frac{1}{3}}, x_{n+\frac{2}{3}}, x_{n+1}$ and collocating (2.2) at $x = x_n, x_{n+\frac{1}{3}}, x_{n+\frac{2}{3}}, x_{n+1}$ lead to a system of equation written below

$$\begin{pmatrix} 1 & x_{n+\frac{1}{3}} & x^2_{n+\frac{1}{3}} & x^3_{n+\frac{1}{3}} & x^4_{n+\frac{1}{3}} & x^5_{n+\frac{1}{3}} & x^6_{n+\frac{1}{3}} \\ 1 & x_{n+\frac{2}{3}} & x^2_{n+\frac{2}{3}} & x^3_{n+\frac{2}{3}} & x^4_{n+\frac{2}{3}} & x^5_{n+\frac{2}{3}} & x^6_{n+\frac{2}{3}} \\ 1 & x_{n+1} & x^2_{n+1} & x^3_{n+1} & x^4_{n+1} & x^5_{n+1} & x^6_{n+1} \\ 0 & 0 & 0 & 6 & 24x_n & 60x_n^2 & 120x_n^3 \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{3}} & 60x^2_{n+\frac{1}{3}} & 120x^3_{n+\frac{1}{3}} \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{2}{3}} & 60x^2_{n+\frac{2}{3}} & 120x^3_{n+\frac{2}{3}} \\ 0 & 0 & 0 & 6 & 24x_{n+1} & 60x^2_{n+1} & 120x^3_{n+1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} y_{n+\frac{1}{3}} \\ y_{n+\frac{2}{3}} \\ y_{n+1} \\ f_n \\ y_{n+\frac{1}{3}} \\ y_{n+\frac{2}{3}} \\ y_{n+1} \end{pmatrix} \tag{2.3}$$

we solve for $a'_j s$ in the above and the resulting value of $a'_j s$ are substituted into (2.1) to yield a continuous implicit hybrid one step method of the form:

$$y(x) = \sum_{j=0}^{k} \alpha_j(x) y_{n+j} + \sum_{wi} \alpha_{wi} y_{n+wi} + h^3 \left[\sum_{j=0}^{k} \beta_j(x) f_{n+j} + \sum_{wi} \beta_{wi} f_{n+wi} \right] \tag{2.4}$$

where $k = 1, wi = 0, \frac{1}{3}, \frac{2}{3}, 1$ and the parameter α_j and β_j, $j = \frac{1}{3}, \frac{2}{3}, 1$ are obtained as

$$\left.\begin{aligned} &\alpha_{\frac{1}{3}} = 3 - \frac{15}{2}t + \frac{9}{2}t^2, \quad \alpha_{\frac{2}{3}} = -3 + 12t - 9t^2, \quad \alpha_1 = 1 - \frac{9}{2}t + \frac{9}{2}t^2 \\ &\beta_0 = h^3\left[\frac{1}{135}t - \frac{31}{540}t^2 + \frac{1}{6}t^3 - \frac{11}{48}t^4 + \frac{3}{20}t^5 - \frac{3}{80}t^6\right] \\ &\beta_{\frac{1}{3}} = h^3\left[-\frac{1}{54} + \frac{25}{216}t - \frac{151}{720}t^2 + \frac{3}{8}t^4 - \frac{3}{8}t^5 + \frac{9}{80}t^6\right] \\ &\beta_{\frac{2}{3}} = h^3\left[-\frac{1}{54} + \frac{43}{540}t - \frac{11}{180}t^2 + \frac{3}{16}t^4 - \frac{3}{10}t^5 - \frac{9}{80}t^6\right] \\ &\beta_1 = h^3\left[\frac{1}{1080}t - \frac{11}{2160}t^2 + \frac{1}{24}t^4 - \frac{3}{40}t^5 - \frac{3}{80}t^6\right] \end{aligned}\right\} \tag{2.5}$$

Evaluating the above at $t = 0$ gives the discrete scheme

$$y_n = 3y_{n+\frac{1}{3}} - 3y_{n+\frac{2}{3}} + y_{n+1} - h^3\left[\frac{1}{54}f_{n+\frac{1}{3}} + \frac{1}{54}f_{n+\frac{2}{3}}\right] \tag{2.6}$$

Where $t = \frac{x - x_n}{h}$ and also note that $\frac{dt}{dx} = \frac{1}{h}$

Differentiating (2.4) once, we obtain

$$y'(x) = \sum_{j=0}^{k} \alpha'_j(x) y_{n+j} + \sum_{wi} \alpha'_{wi} y_{n+wi} + h^3 \left[\sum_{j=0}^{k} \beta'_j(x) f_{n+j} + \sum_{wi} \beta'_{wi} f_{n+wi} \right] \tag{2.7}$$

On evaluating (2.7) at $t = 0, \frac{1}{3}, \frac{2}{3}$ and1, we obtain

$$\left.\begin{aligned} &hy'_n = -\frac{15}{2}y_{n+\frac{1}{3}} + 12y_{n+\frac{2}{3}} + \frac{9}{2}y_{n+1} + h^3\left[\frac{1}{135}f_n + \frac{25}{216}f_{n+\frac{1}{3}} + \frac{43}{540}f_{n+\frac{2}{3}} + \frac{1}{1080}f_{n+1}\right] \\ &hy'_{n+\frac{1}{3}} = -\frac{9}{2}y_{n+\frac{1}{3}} + 6y_{n+\frac{2}{3}} + \frac{3}{2}y_{n+1} + h^3\left[-\frac{1}{1080}f_n + \frac{1}{90}f_{n+\frac{1}{3}} + \frac{29}{1080}f_{n+\frac{2}{3}}\right] \\ &hy'_{n+\frac{2}{3}} = -\frac{3}{2}y_{n+\frac{1}{3}} + \frac{3}{2}y_{n+1} - h^3\left[\frac{1}{1080}f_{n+\frac{1}{3}} + \frac{1}{60}f_{n+\frac{2}{3}} + \frac{1}{1080}f_{n+1}\right] \\ &hy'_{n+1} = \frac{3}{2}y_{n+\frac{1}{3}} - 6y_{n+\frac{2}{3}} + \frac{9}{2}y_{n+1} + h^3\left[\frac{1}{1080}f_n - \frac{1}{270}f_{n+\frac{1}{3}} + \frac{7}{216}f_{n+\frac{2}{3}} + \frac{1}{135}f_{n+1}\right] \end{aligned}\right\} \tag{2.8}$$

Differentiating (2.4) twice, we obtain

$$y''(x)=\sum_{j=0}^{k}\alpha''_j(x)y_{n+j}+\sum_{wi}\alpha''_{wi}\,y_{n+wi}+h^3\left[\sum_{j=0}^{k}\beta''_j(x)f_{n+j}+\sum_{wi}\beta''_{wi}\,f_{n+wi}\right] \qquad (2.9)$$

On evaluating (2.9) at $t=0,\frac{1}{3},\frac{2}{3}$ and 1, we obtain

$$\left.\begin{aligned}
h^2y''_n&=9y_{n+\frac{1}{3}}-18y_{n+\frac{2}{3}}+9y_{n+1}-h^3\left[\frac{31}{270}f_n+\frac{151}{360}f_{n+\frac{1}{3}}+\frac{11}{90}f_{n+\frac{2}{3}}+\frac{11}{1080}f_{n+1}\right]\\
h^2y''_{n+\frac{1}{3}}&=9y_{n+\frac{1}{3}}-18y_{n+\frac{2}{3}}+9y_{n+1}+h^3\left[\frac{11}{1080}f_n-\frac{7}{45}f_{n+\frac{1}{3}}-\frac{23}{120}f_{n+\frac{2}{3}}+\frac{1}{720}f_{n+1}\right]\\
h^2y''_{n+\frac{2}{3}}&=9y_{n+\frac{1}{3}}-18y_{n+\frac{1}{3}}+9y_{n+1}-h^3\left[\frac{1}{270}f_n-\frac{1}{40}f_{n+\frac{1}{3}}+\frac{1}{90}f_{n+\frac{2}{3}}+\frac{1}{1080}f_{n+1}\right]\\
h^2y''_{n+1}&=9y_{n+\frac{1}{3}}-18y_{n+\frac{2}{3}}+9y_{n+1}+h^3\left[\frac{11}{1080}f_n-\frac{2}{45}f_{n+\frac{1}{3}}+\frac{91}{360}f_{n+\frac{2}{3}}+\frac{31}{270}f_{n+1}\right]
\end{aligned}\right\} \qquad (2.10)$$

Collecting (2.6), (2.8) and (2.10) gives the block method which is employed simultaneously to obtain the values for $y_{n+\frac{1}{3}},y_{n+\frac{2}{3}},y_{n+1},y'_{n+\frac{1}{3}},y'_{n+\frac{2}{3}},y'_{n+1},y''_{n+\frac{1}{3}},y''_{n+\frac{2}{3}},y''_{n+1}$ as follows

$$\left.\begin{aligned}
y_{n+\frac{1}{3}}&=y_n+\frac{1}{3}hy'_n+\frac{1}{18}h^2y''_n+h^3\left[\frac{19}{4860}f_n+\frac{7}{2160}f_{n+\frac{1}{3}}-\frac{1}{180}f_{n+\frac{2}{3}}+\frac{1}{3888}f_{n+1}\right]\\
y_{n+\frac{2}{3}}&=y_n+\frac{2}{3}hy'_n+\frac{2}{9}h^2y''_n+h^3\left[\frac{5}{243}f_n+\frac{14}{405}f_{n+\frac{1}{3}}-\frac{1}{135}f_{n+\frac{2}{3}}+\frac{2}{1215}f_{n+1}\right]\\
y_{n+1}&=y_n+hy'_n+\frac{1}{2}h^2y''_n+h^3\left[\frac{1}{20}f_n+\frac{9}{80}f_{n+\frac{1}{3}}+\frac{1}{240}f_{n+1}\right]\\
y'_{n+\frac{1}{3}}&=y'_n+\frac{1}{3}hy''_n+h^2\left[\frac{97}{3240}f_n+\frac{19}{540}f_{n+\frac{1}{3}}-\frac{13}{1080}f_{n+\frac{2}{3}}+\frac{22}{405}f_{n+1}\right]\\
y'_{n+\frac{2}{3}}&=y'_n+\frac{2}{3}hy''_n+h^2\left[\frac{28}{405}f_n+\frac{22}{135}f_{n+\frac{1}{3}}-\frac{2}{135}f_{n+\frac{2}{3}}+\frac{2}{405}f_{n+1}\right]\\
y'_{n+1}&=y'_n+hy''_n+h^2\left[\frac{13}{120}f_n+\frac{3}{10}f_{n+\frac{1}{3}}+\frac{3}{40}f_{n+\frac{2}{3}}+\frac{1}{60}f_{n+1}\right]\\
y''_{n+\frac{1}{3}}&=y''_n+h\left[\frac{1}{8}f_n+\frac{19}{72}f_{n+\frac{1}{3}}-\frac{5}{72}f_{n+\frac{2}{3}}+\frac{1}{72}f_{n+1}\right]\\
y''_{n+\frac{2}{3}}&=y''_n+h\left[\frac{1}{9}f_n+\frac{4}{9}f_{n+\frac{1}{3}}+\frac{1}{9}f_{n+1}\right]\\
y''_{n+1}&=y''_n+h\left[\frac{1}{8}f_n+\frac{3}{8}f_{n+\frac{1}{3}}+\frac{3}{8}f_{n+\frac{2}{3}}+\frac{1}{8}f_{n+1}\right]
\end{aligned}\right\} \qquad (2.11)$$

3.0 Analysis of the Scheme

In this section, we analyze the derived scheme which includes the order and error constant, Consistency zero stability, and convergence of the method.

3.1 Order and error constant

Definition 3.1: The linear operator L and the associate block method are said to be of order p if $C_0=C_1=\cdots=C_p=C_{p+1}=C_{p+2}=0,\quad C_{p+3}\neq 0.$ C_{p+3} is called the error constant and implies that the truncation error is given by $t_{n+k}=C_{p+3}h^{p+3}y^{p+3}(x)+0h^{p+4}$

$$\begin{bmatrix} = \sum_{j=0}^{\infty} \frac{\left(\frac{1}{3}\right)^j}{j!} - y_n - \frac{1}{3}hy'_n - \frac{1}{8}h^2y''_n + \frac{19}{4860}h^3y'''_n - \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \left[\frac{7}{2160}\left(\frac{1}{3}\right) - \frac{1}{810}\left(\frac{2}{3}\right) + \frac{1}{3888}\right] \\ = \sum_{j=0}^{\infty} \frac{\left(\frac{2}{3}\right)^j}{j!} - y_n - \frac{2}{3}hy'_n - \frac{2}{9}h^2y''_n - \frac{5}{243}h^3y'''_n - \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \left[\frac{14}{405}\left(\frac{1}{3}\right) - \frac{1}{135}\left(\frac{2}{3}\right) + \frac{2}{1215}\right] \\ = \sum_{j=0}^{\infty} \frac{(1)^j}{j!} - y_n - hy'_n - \frac{1}{2}h^2y''_n - \frac{1}{20}h^3y'''_n - \sum_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \left[\frac{9}{80}\left(\frac{1}{3}\right) - 0\left(\frac{2}{3}\right) + \frac{1}{240}\right] \end{bmatrix}$$

Comparing the coefficient of h, according to [4], the order p of our method and the error constant are given respectively by $p = [3 \quad 3 \quad 3]^T$ and

$C_{p+3} = [-1.0860 \times 10^{-4} \quad -4.5725 \times 10^{-5} \quad -1.5432 \times 10^{-4}]$

3.2 Consistency of the Method

A numerical method is said to be consistent if the following conditions are satisfied

The order of the method must be greater than or equal to zero to one i.e. $p \geq 1$.

$\sum_{j=0}^{k} \alpha_j = 0$

$\rho(r) = \rho'(r) = 0$

$\rho'''(r) = 3!\sigma(r)$

Where $\rho(r)$ *and* $\sigma(r)$ are first and second characteristics polynomials of our method. According to Skwame, Donald, Kyagya and Sabo [17], the first condition is a sufficient condition for the associated block method to be consistent. Hence our method (2.11) is consistent.

3.3 Zero Stability of the Method

Definition 3.1: The computational method is said to be zero-stable, if the roots z_s, $s = 1, 2, \cdots, k$ of the first characteristics polynomial $\rho(z)$ defined by $\rho(z) = \det\left(zA^{(0)} - E\right)$ satisfies $|z_s| \leq 1$ and every root satisfies $|z_s| = 1$ have multiplicity not exceeding the order of the differential equation, Skwame, Sabo and Mathew [17]. The first characteristic polynomial is given by,

$$\rho(z) = \left| z\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right| = \begin{vmatrix} -z & 0 & 1 \\ 0 & -z & 1 \\ 0 & 0 & 1-z \end{vmatrix} = z^2(z-1)$$

Thus, solving for z in

$$z^2(z-1) < 1 \qquad (2.12)$$

gives $z = 0, 0, 1$. Hence the block method (2.11) is said to be zero stable.

3.4 Convergence of the Block Method

Theorem 4.1: the necessary and sufficient conditions for linear multistep method to be convergent are that it must be consistent and zero-stable. Hence our method (2.11) formulated is consistent, [16].

4.0 Numerical Implementation of the Method

In this section, we test the effectiveness and validity of our newly derived method by applying it to some third order highly stiff and non-stiff initial value problems of the form (1.1), on three test problems. Our result are compared with that of existing methods.

Problem 4.1: Consider the highly non-stiff third order problem

$y'''(x) = 3\cos(x), \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 2,$

with the exact solution: $y(x) = x^2 - 3\sin(x) + 3x + 1$

Source: Tarparki,Gurah and Simon [18]

Table 4.1: Showing the results for problem 4.1

X	**Exact Result**	**Computed Result**	**Error in our Method**	**Error in [18]**
0.1	1.01049975005951554310	1.0439920076148163536	6.5972e-11	2.4800e-07
0.2	1.04399200761481635360	1.0104997500595155431	9.9109e-12	7.3740e-06
0.3	1.10343938001598127470	1.1034393800159812747	2.1387e-10	6.0542e-05
0.4	1.19174497307404852500	1.1917449730740485250	4.9842e-10	2.5479e-04
0.5	1.31172338418739099920	1.3117233841873909992	9.6309e-10	7.7602e-04
0.6	1.46607257981489392840	1.4660725798148939284	1.6495e-09	1.9261e-03
0.7	1.65734693828692683900	1.6573469382869268390	2.5970e-09	4.1505e-03
0.8	1.88793172730143171510	1.8879317273014317151	3.8425e-09	8.3637e-03
0.9	2.16001927111754983460	2.1600192711175498346	5.4197e-09	1.4774e-02
1.0	2.47558704557631048000	2.4755870455763104800	7.3591e-09	2.4702e-02

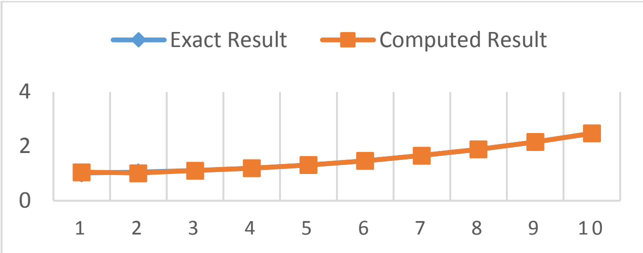

Figure 4.1: Graphical solution of problem 4.1

Problem 4.2: Consider the highly non-stiff third order problem

$$y'''(x) = 3\sin(x), \quad y(0)=1,\ y'(0)=0,\ y''(0)=-2,$$

with the exact solution: $y(x) = 3\cos(x) + \frac{x^2}{2} - 2$

Source: Gbenga, Olaoluwa and Oluyemi [20], Olabode and Yusuf [21], Adeyeye and Omar [3]

Table 4.2: Showing the results for problem 4.2

X	Exact Result	Computed Result	Error in our Method	Error in [19]	Error in [20]	Error in [3]
0.1	0.99001249583407729830	0.99001249583448936532	4.1207e-13	3.3176e-11	1.6592e-10	1.7282e-12
0.2	0.96019973352372489340	0.96019973352767250137	3.9476e-12	2.1230e-10	4.7628e-10	6.3179e-12
0.3	0.91100946737681805890	0.91100946739431976397	1.7502e-11	5.3745e-10	6.2318e-10	1.4295e-11
0.4	0.84318298200865524840	0.84318298206106588483	5.2411e-11	9.2688e-10	1.9913e-10	2.5020e-11
0.5	0.75774768567111814840	0.75774768579541380571	1.2430e-10	1.4510e-09	3.2888e-10	3.8928e-11
0.6	0.65600684472903489170	0.65600684498229039396	2.5326e-10	2.1099e-09	1.2710e-09	5.5360e-11
0.7	0.53952656185346527880	0.53952656231690759749	4.6344e-10	2.8271e-09	4.8465e-09	7.4644e-11
0.8	0.41012012804149626280	0.41012012882431661014	7.8282e-10	3.6574e-09	1.0959e-08	9.6128e-11
0.9	0.26982990481199336940	0.26982990605487988176	1.2429e-09	4.6012e-09	2.0188e-08	1.2002e-10
1.0	0.12090691760441915220	0.12090691948271460664	1.8783e-09	5.5938e-09	3.5396e-08	1.4570e-10

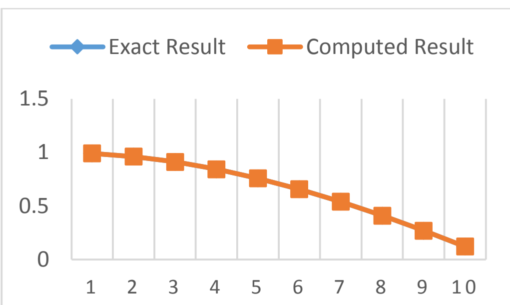

Figure 4.2: Graphical solution of problem 4.2

Problem 4.3: Consider the highly non-stiff third order problem

$$y'''(x) = e^x, \quad y(0)=3,\ y'(0)=1,\ y''(0)=5,$$

with the exact solution: $y(x) = 2 + 2x^2 + e^x$

Source: Olabode and Yusuf [21], Adeyeye and Omar [3], Olabode [22]

Table 4.3: Showing the results for problem 4.3

X	Exact Result	Computed Result	Error in our Method	Error in [20]	Error in [21]
0.1	3.12517091807564762480	3.12517091807909520460	3.4476e-12	9.2453e-10	9.2435e-10
0.2	3.30140275816016983390	3.30140275818359081850	2.3421e-11	1.8398e-09	8.3983e-10
0.3	3.52985880757600310400	3.52985880765388363430	7.7881e-11	2.4200e-09	4.2400e-10
0.4	3.81182469764127031780	3.81182469782790263910	1.8663e-10	5.3587e-09	3.5873e-10
0.5	4.14872127070012814680	4.14872127107163578550	3.7151e-10	7.0013e-10	2.9987e-10
0.6	4.54211880039050897490	4.54211880104726421790	6.5676e-10	3.9051e-10	3.9051e-10
0.7	4.99375270747047652160	4.99375270853965068110	1.0691e-09	6.5295e-09	1.4710e-09
0.8	5.50554092849246760460	5.50554093013084949530	1.6384e-09	2.1508e-08	2.4925e-09
0.9	6.07960311115694966380	6.07960311355406050390	2.3971e-09	3.8843e-08	1.5695e-09
1.0	6.71828182845904523540	6.71828183184049531740	3.3815e-09	6.1541e-08	3.5410e-08

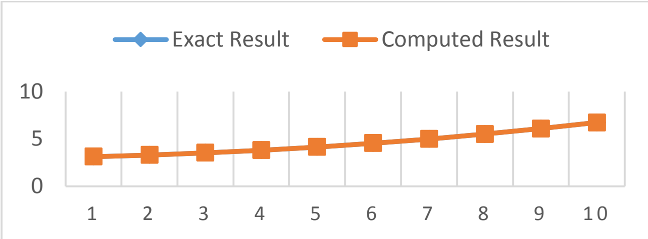

Figure 4.3: Graphical solution of problem 4.3

5 CONCLUSION

We have been able to generate a new approximation scheme of third derivative block method for the direct solution of third order ODEs in this research. The method was derived via interpolation and collocation procedure by introduction of, off-mesh point at both grid and off-grid, using the power series. This new scheme was examined, and was found to be consistent, zero-stable and convergent. Three highly stiff problems were used to test the method and our results show more accurate than the existing methods as exhibited on the tables represented. We further sketched the solution graph of our method and it is evident that the new scheme convergence toward the exact solution.

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