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RANDOM b_i CHANCE-CONSTRAINED STOCHASTIC PROGRAMMING: AN APPLICATION

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Abstract

The study has applied the chance-constrained stochastic programming technique to a building block industry problem, focusing more on when the resources are random quantities. The study is geared towards solving the problems of uncertainty surrounding block production, with a motive of maximizing profit. In applying the approach, some mathematical programming models were specially developed to determine expected profit for when some certain level of uncertainty are considered. Data collected from Dunu block manufacturing industry were the major source of information used in obtaining parameters for the probabilistic model after the study has carefully calculated the means, standard deviations and variances of the various weights of manufacturing materials before fitting them into the models.

***Keywords*:** Chance-Constrained, Stochastic Programming, Lingo Solver, Building Block.

1.1 Introduction

The goal of mathematical programming is to identify an optimal solution to a given problem. Traditionally, it is assumed that all problem data are known precisely, which implies that an optimal solution for the problem is truly the best. Unfortunately, in most cases, problem data cannot be known exactly and instead the data can take a set of values or perhaps can be defined by a probability distribution. This is where stochastic programming comes in.

Stochastic programming is an extension of mathematical programming in which the assumption that all data are known is relaxed; instead, a subset of the parameter values of the problem are characterized by probability distributions. The goal of a mathematical programming problem is to identify an optimal solution, where optimality is defined in terms of a cost function to be minimized or maximized. A kind of stochastic programming model is the chance-constrained stochastic programming [1] It is an operations research approach for optimization under uncertainty when some or all coefficients in a linear program are random variables distributed in accordance with some probability law.

Chance-constrained programming is a branch of stochastic programming which are mostly seen as an unfriendly kind of programming which for their sometimes non-convexity, nonlinearity, discrete variables, and random variables. Charnes and Cooper, formulated the chance constraint in the earlier 1959, and presented the stochastic model as a single constraint (where $m = 1$). The model formulated by Charnes and Cooper [2], had a fixed left-hand side. They went further to show that the problem can be reformulated as a deterministic nonlinear programming problem which was achieved by taking the inverse of the distribution of the random right hand side of the problem.

Instead of requiring feasibility almost surely, a chance-constraint within a stochastic program must be satisfied at least with probability. Chance-constraint introduces dependency into this concept, requiring that a subset of constraints in the formulation are satisfied at least with probability.

The mixture of these materials makes it almost impossible for the industry to determine the various profit she makes from each block product considering also the damages incurred after the consumption of some kilograms of cement and a truck of sand (sharpsand, stonedust, 3/8 stones) and as such making it difficult to determine the brand or product that accrues the largest percentage of the company's annual profit.

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Chance-constraints are used to model systems for which a certain quality of service is required, or for when a problem has extreme cases for which satisfying the chance-constraints for all possible parameter values is either too expensive or impossible.

This study was undertaken to investigate the usefulness of considering the possibility of randomness in the quantity of raw material used in a manufacturing company. The study uses the building block manufacturing company as a case study, where we have taken the rigor to investigate and measure materials used for making a piece of block of various brands. It is believed that the study can provide the managements of building block industries with a suitable mathematical model capable of increasing productivity and optimality.

1.2 Valuable Literatures

Ahmed [3], developed convex relations of chance-constrained optimization problems in order to obtain lower bounds on the optimal value. Unlike existing statistical lower bounding techniques, the approach was designed to provide deterministic lower bounds. Wang et al. [4], applied chance-constrained programming on project scheduling problem in an uncertain environment where the duration time of each activity is an uncertain variable. Cooper et al. [5], developed models using chance-constrained programming formulations for treating congestion in data envelopment analysis (DEA). Gali et al. [6], applied chance-constrained programming to enhance decision-making by Queensland barley growers. The model was designed to help Queensland barley growers make varietal and agronomic decisions in the face of changing product demands and volatile production conditions. Ruszczynski et al. [7], considered chance-constrained stochastic programming problems allowing for the left-hand side of the constraints to be random.

1.3 Aim of the Study

The aim of this research work is to formulate a Chance-constrained stochastic programming model to assist in determining block products with optimal profit per truck of sand (sharpsand, stonedust, 3/8 stones).

1.4 Motivation

Some scholars are certainly already familiar with deterministic optimization methods such as the linear programming. The interest for carrying out this research work on chance-constrained programming can come from different sources. Our interest stems from our curiosity to apply this unique programming technique in a real life scenario like the block industry problem considered in this study. Technically, stochastic programs are much more complicated than the corresponding deterministic programs. Hence, at least from a practical point of view, there must be very good reasons to turn to the stochastic programming methods since almost all real world events are characterized with some form of uncertainty.

Deterministic models may certainly produce good solutions for certain data sets, but there is generally no way you can conclude that they are good without comparing them to solutions of stochastic programs. In many cases, solutions to deterministic programs are very misleading, so the use of most suitable programming technique to solve the block industry problem created in this book has been necessitated.

2.1 Methodology

1. Method of Data Collection

Data used in this study was collected from Dunu block manufacturing industry and comprises the various weights of material shipments made to the industry. The research was able to record about eight (8) weight of each delivery of the three major material used by the block industry (sharpsand, stonedust, and 3/8 stones). The deliveries were made by a kind of tipper truck called the 10 tyre truck which usually are able to carry more than 20 tons of sand.

2. Derivation of the General Chance-Constrained Stochastic

When only $\boldsymbol{b_i}$ are random variables: let $\bar{b}_i$ and $var\{b_i\}$ denote the mean and variance of the normally distributed random variable $\bar{b}_i$. The constraints can be restated as:

$$Maximize\ Z = \sum_{j=1}^{n} c_j x_j \quad (1.1)$$

$$P\left\{\sum_{j=1}^{n} a_{ij} x_j \le b_i\right\} \ge \alpha_i \quad (1.2)$$

This is followed by subtracting $\frac{\bar{b}_i}{\sqrt{var\{b_i\}}}$ from both sides of the LHS of (1.2).

$$P\left\{\sum_{j=1}^{n} \frac{a_{ij} x_j - \bar{b}_i}{\sqrt{var\{b_i\}}} \le \frac{b_i - \bar{b}_i}{\sqrt{var\{b_i\}}}\right\} \ge \alpha_i, \qquad i = 1,2 \ldots.. m \quad (1.3)$$

Where $\frac{b_i - \bar{b}_i}{\sqrt{var\{b_i\}}}$ is a standard normal variable with zero mean and unit variance. The inequalities in equation (1.3) can be rewritten as:

$$P\left\{\sum_{j=1}^{n} \frac{a_{ij} x_j - \bar{b}_i}{\sqrt{var\{b_i\}}} \le \frac{b_i - \bar{b}_i}{\sqrt{var\{b_i\}}}\right\} \le 1 - \alpha_i, \qquad i = 1,2 \ldots.. m \quad (1.4)$$

If K_i represents the standard normal variate at which;

$$\Phi(K_i) = 1 - \alpha_i \quad (1.5)$$

The constraints in equation (1.4) can be expressed as:

$$\Phi\left\{\sum_{j=1}^{n} \frac{a_{ij} x_j - \bar{b}_i}{\sqrt{var\{b_i\}}}\right\} \le \Phi(K_i), \qquad i = 1,2 \ldots.. m \quad (1.6)$$

From (1.6), we get the below (1.7) or (1.8):

$$\sum_{j=1}^{n} \frac{a_{ij}x_j - \bar{b}_i}{\sqrt{var\{b_i\}}} \leq K_i, \qquad i = 1,2 \ldots ..m \qquad (1.7)$$

$$\sum_{j=1}^{n} a_{ij}x_j - \bar{b}_i + K_i\sqrt{var\{b_i\}} \leq 0, \qquad i = 1,2 \ldots ..m \qquad (1.8)$$

The Chance-Constrained model for when only b_i are random variables is derived below as:

$$\left.\begin{array}{l} Maximize\ z = \sum_{j=1}^{n} c_j x_j \\ subject\ to: \\ \sum_{j=1}^{n} a_{ij}x_j + K_i\sqrt{var\{b_i\}} \leq \bar{b}_i, i = 1,2, \ldots .m \\ where\ \ x_j \geq 0, and\ \ j = 1,2, \ldots \ldots .n \end{array}\right\} \qquad (1.9)$$

z is the objective function with the largest possible total farm gross margin.
c_j is the profit from a unit of the jth activity.
x_j is the level of the ith resourse required to produce one unit of the jth activity.
a_{ij} is the quantity of ith resource required to produce one unit of the jth activity.
$\bar{b_l}$ is the mean level of the ith resource or constraint.
α_i is the minimum probability of meeting the ith constraint.
K_i is the value of the standard normal variate corresponding to the probability α_i

3. Data and Some Derived Value Tables

Table 1: Standard distribution of various weight of material (kg)

Distribution	Sharpsand	Stonedust	3/8 stone
Mean	19900	19060	19340
Standard deviation	254.951	748.227	612.245
Variance	65000	559843.75	374843.75

Table 2: Costs of material (Sand and stone)

	Sharpsand {19.9tons}	Stonedust {19.06tons}	3/8stone {19.34tons}
Cost per ton	—	₦1800	₦2100
Transportation per truck	—	₦18000	₦18000
Total cost	₦25000	₦52300	₦58600

Table 3: Weight of cement in each block product in kg

	SCB 9"	SCB 6"	SDB 9"	SDB 6"	SDB 4"	CCB 9"	CCB 6"
Mean	1.227	1.024	1.252	1.056	0.88	1.19	0.96
SD	0.052	0.027	0.044	0.038	0.022	0.030	0.016

Table 4: Mean weight and standard deviation of sand materials in each block product.

	SCB 9"	SCB 6"	SDB 9"	SDB 6"	SDB 4"	CCB 9"	CCB 6"
Mean	22.56	14.08	30.87	20.16	14.12	35.33	23.38
SD	0.20	0.34	0.27	0.20	0.34	0.24	0.23

Table 5: A distribution showing cement consumption per truck.

	Truck of Sharpsand	Truck of Stonedust	Truck of 3/8 stone
Mean	1252.206	979.983	715.969
Standard deviation	185.593	169.913	71.935
Variance	34444.826	28870.519	5174.571

Table 6: Weights of Material and Number of blocks per truck

Block Brands	Required Cement for a Sand Truck (kg)	Average Blocks per Truck	Average Block Weights (kg)	Average Blocks per Cement bag
SCB 9"	1076.04	882	23.70	41
SCB 6"	1446.9	1413	15.04	49
SDB 9"	772.5	617	32.06	40
SDB 6"	997.9	945	21.14	48
SDB 4"	1188	1350	14.94	57
CCB 9"	650.9	547	36.46	43
CCB 6"	793.9	827	24.26	53

Table 7: Material costs and profit per block brand

	Cement Cost (₦)	Cost of Sharpsand Stonedust & 3/8 Stone (₦)	Labour Cost (₦)	Average Production Cost (₦)	Selling Price (₦)	Average Profit (₦)
SCB 9"	35.38	28.43	15	78.81	155	76.19
SCB 6"	29.70	17.74	14	61.44	130	68.56
SDB 9"	36.31	84.89	18	139.20	200	60.78
SDB 6"	30.69	55.54	25	111.23	160	49.77
SDB 4"	25.50	38.79	14	78.29	120	40.71
CCB 9"	34.51	107.05	31	172.56	235	62.45
CCB 6"	31.84	93.77	29	154.61	205	50.39

3.1 Results

Tableau One: When only the right-hand-sides are random.

Block brands	Materials required for block production							
	Cement		Sharpsand		Stonedust		3/8 stone	
	Mean $\bar{b}_1$	SD σ	Mean $\bar{b}_2$	SD σ	Mean $\bar{b}_3$	SD σ	Mean $\bar{b}_4$	SD σ
Sandcrete blocks	1252.21	185.59	19900	254.95				
Stonedust blocks	979.98	169.91			19060	748.28		
Concrete blocks	715.97	71.94					19340	612.25

Where the profit is gotten from table 7.

In generating a model for this scenario, a general model just like can be gotten for when a_{ij} are random has not been achieved here. This is because of the difficulty in finding a collective mean, standard deviation and variance for cement consumption per truck for all the products. So, the research has generated separate models that would optimize profit for the three block brands sharing the same production materials. We note that the sandcrete block brand has two product, the stanedust - three, and concrete block brand has two products.

Now, we commence with the sandcrete model.

3.1 Models for Sandcrete Blocks

The objective function for this model is:

$Z = 76.19x_1 + 68.56x_2$

Below, we derive the cement and sharpsand constraints using (1.8) the above equation.

1. Cement constraint {kg}

$$\sum_{j=1}^{2} a_{ij}x_j \leq \bar{b}_i + K_i\sqrt{var\{b_i\}}, i = 1 \quad (2.2)$$

We expand to have: $a_{11}x_1 + a_{12}x_2 \leq \bar{b}_1 - K_i\sqrt{var\{b_1\}}$ (2.3)

Thus;

$1.227x_1 + 1.024x_2 \leq 1252.21 - K_i\sqrt{34444.83}$

$1.227x_1 + 1.024x_2 \leq 1252.21 - K_i(185.593)$

2. Sharpsand constraint {kg}

Considering equation (1.8) when $j = 1,2 \quad and \quad i = 2,$ then,

$a_{21}x_1 + a_{22}x_2 \leq \bar{b}_2 + K_i\sqrt{var\{b_2\}}$

Such that;

$22.56x_1 + 14.08x_2 \leq 19990 - K_i(254.951)$
Hence, the model for sandcrete block becomes:
$Maximize\ Z = 76.19x_1 + 68.56x_2$
$subject\ to$:
$1.227x_1 + 1.024x_2 \leq 1252.21 - K_i(185.593)$
$22.56x_1 + 14.08x_2 \leq 19990 - K_i(254.951)$
$where \quad x_1 \geq 0, x_2 \geq 0$
For K_i, we add the product collections from each brand and take the average:
SCB 9" = 882, SCB 6" = 1413
$$\frac{882 + 1413}{2} = 1148\ blocks$$
$$\frac{15}{1148} \times 100\% = 1.306,\ which\ implies\ (100 - 1.306)\%\ success,\ K_1 = 0.987.$$

Table 9: Solution to the models provided using Lingo linear programming software:

	15 block damages $K_1 = 0.987$	10 block damages $K_2 = 0.991$	5 block damages $K_3 = 0.996$	0 block damages $K_4 = 1.000$
x_1	812	773	829	790
x_2	71	117	49	95
Profit (₦)	66379.04	66331.39	66275.95	66228.30

4.1 Model for Stonedust blocks

The objective function for this model is:
$Z = 60.78x_3 + 49.77x_4 + 40.71x_5$

1. Cement constraint {kg}

$$\sum_{j=3}^{5} \bar{a}_{ij}x_j \leq \bar{b}_i + K_i\sqrt{var\{b_i\}}, i = 1 \quad (2.4)$$

We expand to have: $\bar{a}_{13}x_3 + \bar{a}_{14}x_4 + \bar{a}_{15}x_5 \leq \bar{b}_1 + K_i\sqrt{var\{b_1\}}$ (2.5)
Thus;
$1.252x_3 + 1.056x_4 + 0.878x_5 \leq 979.98 + K_i\sqrt{28870.52}$
$1.252x_3 + 1.056x_4 + 0.878x_5 \leq 979.98 + K_i(169.913)$

2. Stonedust constraint {kg}

Considering equation (2.4) when $i = 2$
To have,
$\bar{a}_{23}x_3 + \bar{a}_{24}x_4 + \bar{a}_{25}x_5 \leq \bar{b}_2 + K_i\sqrt{var\{b_2\}}$
$30.87x_3 + 20.16x_4 + 14.12x_5 \leq 19060 + K_i\sqrt{559843.75}$
$30.87x_3 + 20.16x_4 + 14.12x_5 \leq 19060 + K_i(748.227)$
Hence, the model for stonedust block becomes:
$Maximize\ Z = 60.78x_3 + 49.77x_4 + 40.71x_5$
$subject\ to$:
$1.252x_3 + 1.056x_4 + 0.878x_5 \leq 979.98 + K_i(169.913)$
$30.87x_3 + 20.16x_4 + 14.12x_5 \leq 19060 + K_i(748.227)$
$where \quad x_3 \geq 0, x_4 \geq 0, x_5 \geq 0$
For K_i, we add the product collections from each brand and take the average:
SDB 9" = 617, SDB 6" = 945, and SDB 4" = 1350
$$\frac{617 + 945 + 1350}{3} = 971\ blocks$$
$$\frac{15}{971} \times 100\% = 1.545,\ which\ implies\ (100 - 1.545)\%\ success,\ K_1 = 0.985.$$

Table 10: Solution to the models provided using Lingo linear programming software:

	15 block damages $K_1 = 0.985$	10 block damages $K_2 = 0.990$	5 block damages $K_3 = 0.995$	0 block damages $K_4 = 1.000$
x_3	466	480	409	411
x_4	79	35	278	274
x_5	166	198	6	7
Profit (₦)	39013.17	38976.93	38939.34	38902.53

4.2 Mode for Concrete blocks

The objective function for this model is

$Z = 62.45x_6 + 50.39x_7$

1. Cement constraint {kg}

$$\sum_{j=6}^{7} \bar{a}_{ij}x_j \leq \bar{b}_i + K_i\sqrt{var\{b_i\}}, i = 1 \quad (2.6)$$

We expand to have: $\bar{a}_{16}x_6 + \bar{a}_{17}x_7 \leq \bar{b}_1 + K_i\sqrt{var\{b_1\}}$ (2.7)

Thus;

$1.19x_6 + 0.96x_7 \leq 715.97 + K_i\sqrt{5174.57}$

$1.19x_6 + 0.96x_7 \leq 715.97 + K_i(71.935)$

2. 3/8 stone constraint {kg}

Considering equation (2.6) when $i = 2$

$\bar{a}_{26}x_6 + \bar{a}_{27}x_7 \leq \bar{b}_2 + K_i\sqrt{var\{b_2\}}$

Hence,

$35.33x_6 + 23.38x_7 \leq 19340 + K_i\sqrt{374843.75}$

$35.33x_6 + 23.38x_7 \leq 19340 + K_i(612.245)$

Hence, the model for concrete block becomes:

$Maximize\ \boldsymbol{Z} = 62.45x_6 + 50.39x_7$

$subject\ to$:

$1.19x_6 + 0.96x_7 \leq 715.97 + K_i(71.935)$

$35.33x_6 + 23.38x_7 \leq 19340 + K_i(612.245)$

$where \quad x_6 \geq 0, x_7 \geq 0$

For K_i, we add the product collections from each brand and take the average:

$CCB\ 9" = 547, and\ \ CCB\ 6" = 827$

$\frac{547 + 827}{2} = 687\ blocks$

$\frac{15}{687} \times 100\% = 2.18\%,\ which\ implies\ \ (100 - 2.18)\%\ success,\ K_1 = 0.978.$

Table 11: Solution to the models provided using Lingo linear programming software:

	15 block damages $K_1 = 0.978$	10 block damages $K_2 = 0.985$	5 block damages $K_3 = 0.993$	0 block damages $K_4 = 1.000$
x_6	23	25	35	37
x_7	644	641	628	625
Profit (₦)	33886.22	33859.96	33829.41	33803.15

4.3 Summary

Chance-constrained stochastic programming technique is a very beautiful mathematical programming approach just as we have seen in this study. The technique allows for consideration of uncertainties which can occur in a real world. The block industry problems considered in this work is a unique case which at the best of our knowledge has not been considered previously. The results generated for the problems for when only the right-hand-side of the models are random, represents the beauty of the study considering that it dealt only with the resource variables. This is adequate for industries in large scale manufacturing and provides them with an insight of expected profit when multiple levels of damages are incurred during production.

4.4 Conclusion

The study has obtained numerous results for each probability level as obtained from Dunu block industry and the chance-constrained stochastic models generated for the problems have been solved using the Lingo solver with results displayed in the study. The various weights of materials obtained from the industry have been carefully fitted into the models as required by the procedures for generating a stochastic programming models. The study has noticed that the model tends to give more priority to one of the blocks from each brand which isn't appropriate for business especially minding the fact that companies would want to market all products that are in high demand. But, this will provide a good guide for the industry in planning and realizing products of better profit.

Future work: The results of the study can further be enhanced by applying ranges to the level of changes that can be made to the coefficients of the linear models. This can be done by the application of the parametric programming technique.

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